Waterloo Workshop in Computer Algebra 2011, W80

Celebrating the achievements and influence of Herbert S. Wilf on the occasion of his 80th birthday

Algorithms and Complexity
Mathematics for the Physical Sciences
generatingfunctionology
A=B

May 26 - 29, 2011

Room N 1001, Faculty of Science Building (WLU)

Wednesday, May 25

17:00		Registration opens, Room N 1001, Faculty of Science Building
18:00		**RECEPTION**

Thursday, May 26

9:00		**Registration, coffee**
9:30	Max Blouw	President, Wilfrid Laurier University, Opening remarks
9:45	George Andrews	Partition Function Differences, and Anti-Telescoping
10:25		**Coffee break**
10:40	Doron Zeilberger	Automatic Generation of Theorems and Proofs on Enumerating Consecutive WILF-classes
11:20	Carla Savage	Generalized Lecture Hall Partitions and Eulerian Polynomials
12:00		**Lunch**
13:00	Robin Pemantle	Zeros of complex polynomials and their derivatives
13:40	Bruce Sagan	Mahonian Pairs
14:20		**Coffee break**
14:40	Gert Almkvist	Ramanujan-like formulas for $1/\pi^2$ and String Theory
15:20	Aviezri Fraenkel	What's a question to Herb Wilf's answer?
16:00	John Stembridge	A finiteness theorem for W-graphs

Friday, May 27

9:00	Ian Goulden	Combinatorics and the KP hierarchy
9:40	Andrew Odlyzko	Primes, graphs, and generating functions
10:20		**Coffee break**
10:40	Jeffrey Shallit	50 Years of Fine and Wilf
11:20	Eugene Zima	Synthetic division in the context of indefinite summation
12:00		**Lunch**
13:00	Volker Strehl	Aspects of a combinatorial annihilation process
13:40	Victor Moll	p-adic valuations of sequences: examples in search of a theory
14:20	Michelle Wachs	Unimodality of q-Eulerian Numbers and p,q-Eulerian Numbers
15:00		**Coffee break**
15:20	Curtis Greene	Some Posets Related to Muirhead's, Maclaurin's, and Newton's Inequalities
16:00	Herb Wilf	Two exercises in combinatorial biology
18:30		**CONFERENCE BANQUET, Paul Martin Center (PMC)**

Saturday, May 28

9:00	Rod Canfield	The Asymptotic Hadamard Conjecture
9:40	Ira Gessel	On the WZ method
10:20		**Coffee break**
10:40	Andrew Granville	More combinatorics and less analysis: A different approach to prime numbers
11:20	Joan Hutchinson	Some challenges in list-coloring planar graphs
12:00		**Lunch**
13:00	Marko Petkovsek	On enumeration of structures with no forbidden substructures
13:40	Richard Stanley	Products of Cycles
14:20	Ronald Graham	Joint statistics for permutations in S_n and Eulerian numbers
15:00		**Coffee break**
15:20	Christian Krattenthaler	Cyclic sieving for generalised non-crossing partitions associated to complex reflection groups
16:00	Peter Paule	Proving strategies of WZ-type for modular forms

Sunday, May 29

9:00	Miklos Bona	Permutations as Genome Rearrangements
9:40	David Jackson	Enumerative aspects of cactus graphs
10:20		**Coffee break**
10:40	Sylvie Corteel	Enumeration of staircase tableaux
11:20		**CLOSING**

Computer Algebra Research Group
of
Wilfrid Laurier University

http://www.cargo.wlu.ca/W80/

WWCA 2011, W80 Abstracts

Author: Gert Almkvist
Affiliation: University of Lund, Sweden
Title: Ramanujan-like formulas for $\frac{1}{\pi^2}$ and String Theory
Abstract: This is joint work with Jesus Guillera, Zaragoza. Using the Gromov-Witten potential from String Theory we design a Maple programme to find formulas for $\frac{1}{\pi^2}$. One result is the new formula

$$\frac{1}{\pi^2} = \frac{32}{3}\sum_{n=0}^{\infty} \frac{(6n)!}{n!^6}(532n^2+126n+9)\frac{1}{1000^{2n+1}}$$

which can be used to compute an arbitrary decimal digit of $\frac{1}{\pi^2}$ without computing the earlier digits.

Author: George E. Andrews
Affiliation: Pennsylvania State University, USA
Title: Partition Function Differences, and Anti-Telescoping
Abstract: For decades partition function differences have been studied. These include a famous problem of Henry Alder posed in the 1950's and solved only recently by Yee, Oliver et al.. In 1978, Szekeres and Richmond partially solved a problem of this type concerning the Rogers-Ramanujan continued fraction. Unknown to them, the problem had essentially been solved by Ramanujan in the Lost Notebook. In this talk, I will begin with the history of such problems. I will conclude with some observations on a method, Anti-Telescoping, for treating some such problems. Here is a typical example of the questions posed. The late Leon Ehrenpreis asked in 1987 if one could prove that the number of partitions of n into parts congruent to 1 or 4 *mod* 5 is always at least as large as the number with parts congruent to 2 or 3 *mod* 5 WITHOUT using the Rogers-Ramanujan identities. Subsequently Baxter and I gave a "sort of" solution to the problem, and Kevin Kadell gave a complete solution in 1999. We shall describe how Anti-Telescoping treats this problem.

Author: Miklos Bona
Affiliation: University of Florida, USA
Title: Permutations as Genome Rearrangements
Abstract: Given a permutation written in the one-line notation, such as 3147526, it is natural to ask how many block transpositions (interchanges of two adjacent substrings) are needed to turn this permutation into the increasing one. This has turned out to be a surprisingly difficult problem, and a long-standing conjecture has recently been disproved in this area. If, on the other hand, we are allowed to

interchange any two blocks, then the best sorting algorithm is known. The average number of necessary block interchanges has recently been computed, using some very unexpected tools from remote-looking areas of mathematics. In this talk, we will review the results and open problems of these two families of questions, and suggest another interesting open problem connected to them. We will say a few words about the biological motivation of these questions, and discuss some of their variations as well. No previous knowledge of sorting algorithms is necessary, and the talk will be accessible to students.

AUTHORS: Rodney Canfield
AFFILIATION: University of Georgia, USA
TITLE: The Asymptotic Hadamard Conjecture
ABSTRACT: The Hadamard Conjecture states that for every integer n which is divisable by 4 there is an $n \times n$ matrix over $\{\pm 1\}$ whose rows are pairwise orthogonal. The first value of n in question is 668. Let H_{nt} equal the number of $n \times t$ matrices over $\{\pm 1\}$ whose rows are pairwise orthogonal. The Asymptotic Hadamard Conjecture gives an asymptotic formula for H_{nt}. The conjecture has been proven by de Launey and Levin (2010) for $t > n^{12+\epsilon}$. We are attempting to extend the range of validity for the formula.

AUTHOR: Sylvie Corteel
AFFILIATION: Universite Paris 7, France
TITLE: Enumeration of Staircase Tableaux
ABSTRACT: Staircase tableaux were recently introduced by Williams and the speaker to capture the combinatorics of the Partially Asymmetric Self Exclusion Process and the moments of the Askey Wilson polynomials. In this talk the speaker will focus on the enumeration of staircase tableaux at various specializations of the parameterizations; for example, we will see how to obtain the Catalan numbers, Fibonacci numbers, Eulerian numbers, the number of permutations, and the number of matchings.

AUTHOR: Aviezri S. Fraenkel
AFFILIATION: Weizmann Institute of Science, Israel
TITLE: What's a question to Herb Wilf's answer?
ABSTRACT: An answer doesn't determine the matching question uniquely. Recently, Herb Wilf and Warren Ewens wrote their revolutionary "There's plenty of time for evolution", in which they refuted the refutation of Darwin's theory that exponential time is needed for evolution. They argue that evolution progresses in parallel, not in series, thereby reducing evolution time drastically. While reading this interesting treatise, I asked myself what other theories (population genetics, phylogenetics, evolutionary biology, geology, geosciences paleobiology, ...), could be consistent with their theory. I think that I was led to this question since I recently became interested in "inverse problems" in the area of combinatorial games. Roughly speaking, given a winning strategy, what's a game that has the given winning strategy? I compounded this with another question. The winning strategy of Wythoff type games on two piles of tokens is usually given in the form of two complementary sequences of integers. What happens if the sequences are not complementary? We give some questions to these answers.

Author: Ira Gessel
Affiliation: Brandeis University, USA
Title: On the WZ Method
Abstract: It is well known that the WZ method of Wilf and Zeilberger gives an efficient way of proving hypergeometric series identities, but each example of the method is usually presented as an isolated application. I will explain how nearly all examples of the WZ method may be associated with special cases of the classical hypergeometric summation formulas of Gauss, Pfaff-Saalschutz, and Dougall.

Author: Ian Goulden
Affiliation: University of Waterloo, Canada
Title: Combinatorics and the KP hierarchy
Abstract: Maps in an orientable surface of arbitrary genus and branched covers of the sphere can both be represented by factorizations in the symmetric group, in which the subgroup generated by the factors acts transitively on the underlying symbols (these are called "transitive factorizations"). The generating series for a large class of transitive factorizations satisfies the KP hierarchy. We shall discuss the KP hierarchy and a new algebraic combinatorial proof of the fundamental result that relates Schur function expansions of a series and the Plucker relations. As an application, we give a recurrence for triangulations of a surface of arbitrary genus obtained from the simplest partial differential equation in the KP hierarchy. The recurrence is very simple, and we do not know a combinatorial interpretation of it, yet it leads to precise asymptotics for the number of triangulations with n edges, in a surface of genus g.

Author: Ronald Graham
Affiliation: UCSD, USA
Title: Joint statistics for permutations in S_n and Eulerian numbers
Abstract: In this talk I will describe some recent results concerning the connection between the bubblesort sorting algorithm and certain integer sequences used to analyze patterns that arise in juggling. The analysis leads to new results on the joint distribution of the descent and maximum drop statistics of a permutation, as well as a new class of identities for the classical Eulerian numbers.

Author: Andrew Granville
Affiliation: Université de Montreal, Canada
Title: More combinatorics and less analysis: A different approach to prime numbers
Abstract: Since Riemann's memoir 150 years ago, the main approach to studying the distribution of prime numbers has come from the study of the complex zeros of the analytic continuation of the Riemann zeta function. Indeed many regard the study of the seeming combinatorial problem of counting primes as tautologically the same as the study of zeta(s), as seems to be indicated by Riemann's brilliant explicit formula. In this talk we introduce the "pretentious approach" of Soundararajan and the speaker to the distribution of prime numbers, which is quite analogous to key ideas in additive combinatorics/number theory and does not rest on the study of analytic continuations.

AUTHOR: Curtis Greene
AFFILIATION: Haverford College, USA
TITLE: Some Posets Related to Muirhead's, Maclaurin's, and Newton's Inequalities
ABSTRACT: Many classical inequalities such as those in title can be extended to familiar combinatorial symmetric function families such as the elementary, homogeneous, power sum, and Schur polynomials. All known instances of these extended inequalities hold in a strong form ("Y-positivity"), and these results are apparently new. Our analysis has involved certain posets which are of some interest in their own right, including the "double-majorization order" on partitions and the lattice of 2-rowed standard Young tableaux. We will survey some of these techniques, and report on some recent results and open problems. This is joint work with Mark Skandera and Jonathan Lima.

AUTHOR: Joan Hutchison
AFFILIATION: Macalester College, USA
TITLE: Some Challenges in List-Colouring Planar Graphs
ABSTRACT: A graph G is said to be L-list-colorable when each vertex v is assigned a list $L(v)$ of colors and G can be properly colored so that each v receives a color from $L(v)$. Typically the lists L may vary from vertex to vertex. A graph is said to be k-list-colorable when it can be L-list-colored whenever every list $L(v)$ contains at least k colors. A celebrated theorem of C. Thomassen proves that every planar graph can be 5-list-colored. An unresolved question of M.O. Albertson asks whether there is a distance $d > 0$ such that whenever a set P of vertices of a planar graph G are precolored and are mutually at distance at least d from one another, the precoloring extends to a 5-list-coloring of G. In this talk we give some partial affirmative answers to Albertson's question and investigate the extent to which Thomassen's theorem and Albertson's question are best possible. This talk includes joint work with co-authors M.O. Albertson, M. Axenovich, and M.A. Lastrina.

AUTHOR: David M.R. Jackson
AFFILIATION: University of Waterloo, Canada
TITLE: Enumerative Aspects of Cactus Graphs
ABSTRACT: I shall discuss enumerative aspects of cactus graphs in the context of a formal analogue of the Legendre transform. Extensions to the m-th order Legendre transform will be considered, as well as the impact of this work on other areas. Attention will be confined to the univariate case, although extension to the multivariate case seems feasible.

AUTHOR: Christian Krattenthaler
AFFILIATION: University of Vienna, Austria
TITLE: Cyclic Sieving for Generalised Non-Crossing Partitions Associated to Complex Reflection Groups
ABSTRACT: Cyclic sieving is a(n enumerative) phenomenon formulated by Reiner, Stanton and White. Bessis and Reiner proposed two conjectures on cyclic sieving phenomena for the generalised non-crossing partitions associated to complex reflection groups of Armstrong and Bessis. I shall first explain what cyclic sieving and these generalised non-crossing partitions are about, and then report the main ideas of a proof of the above two conjectures. Part of this work is in collaboration with Thomas Muller.

AUTHOR: Victor H. Moll
AFFILIATION: Tulane University, USA
TITLE: p-adic Valuations of Sequences: Examples in Search of a Theory
ABSTRACT: In this lecture we present a variety of results and problems related to the p-adic valuation of classical sequences. Examples include Stirling numbers and the ASM-numbers that count the number of alternating sign matrices. An attempt to form a general theory will be discussed.

AUTHOR: Andrew Odlyzko
AFFILIATION: University of Minnesota, USA
TITLE: Primes, Graphs and Generating Functions
ABSTRACT: Herb Wilf had extensive influence on the development of many fields of mathematics, both through his own research, and through asking penetrating questions. Some personal reminiscences of his influence and his collaborations will be presented.

AUTHOR: Peter Paule
AFFILIATION: Research Institute for Symbolic Computation, Johannes Kepler University Linz, Austria
TITLE: Proving strategies of WZ-type for modular forms
ABSTRACT: In the context of WZ-theory Lily Yen and others have shown how hypergeometric identities can be proven by checking finitely many cases only. The talk, being joint work with Silviu Radu (RISC), discusses the algorithmic application of similar ideas, including the role of recurrences, to problems involving modular forms. One of the illustrating examples is a new proof of Ramanujan's celebrated partition congruences for powers of 11.

AUTHOR: Robin Pemantle
AFFILIATION: University of Pennsylvania, USA
TITLE: Zeros of Complex Polynomials and their Derivatives
ABSTRACT: The zeros of f' are known to lie in the convex hull of the zeros of f. Often one can say much more. When the zeros of f are independent random picks from a distribution, it turns out that the zeros of f' have the same distribution in the limit. Under further assumptions, nearly all of the zeros of f' are matched to corresponding zeros of f. This is joint work with I. Rivin.

AUTHOR: Marko Petkovsek
AFFILIATION: University of Ljubljana, Slovenia
TITLE: On Enumeration of Structures with no Forbidden Substructures
ABSTRACT: Many interesting classes of combinatorial structures are defined by restricting some general class of structures to those structures that avoid certain "forbidden" substructures. Examples include words avoiding forbidden subwords or subsequences, permutations avoiding forbidden patterns, matrices avoiding forbidden submatrices, graphs avoiding forbidden subgraphs, induced subgraphs, minors, or topological minors. We will try to look at the abundance of enumeration problems (solved and unsolved) presented by such classes.

AUTHORS: Bruce E. Sagan
AFFILIATION: Michigan State University, USA
TITLE: Mahonian Pairs
ABSTRACT: We introduce the notion of a Mahonian pair. Consider the set, $\mathbb{P}^*$, of all words having the positive integers as alphabet. Given finite subsets $S, T \subset \mathbb{P}^*$, we say that (S, T) is a *Mahonian pair* if the distribution of the major index, maj, over S is the same as the distribution of the inversion number, inv, over T. So the well-known fact that maj and inv are equidistributed over the symmetric group, $\mathfrak{S}_n$, can be expressed by saying that $(\mathfrak{S}_n, \mathfrak{S}_n)$ is a Mahonian pair. We investigate various Mahonian pairs (S, T) with $S \neq T$. Our principal tool is Foata's fundamental bijection $\phi : \mathbb{P}^* \to \mathbb{P}^*$ since it has the property that maj w = inv $\phi(w)$ for any word w. We consider various families of words associate with Catalan and Fibonacci numbers. Various other ideas come into play such as the ranks and Durfee square size of integer partitions, the Catalan triangle, and various q-analogues.

AUTHOR: Carla D. Savage
AFFILIATION: NCSU, USA
TITLE: Generalized Lecture Hall Partitions and Eulerian Polynomials
ABSTRACT: Lecture hall partitions, introduced by Bousquet-Melou and Eriksson in 1997, are nonnegative integer sequences $(x_1, x_2, ..., x_n)$ satisfying $x_i/i <= x_{i+1}/(i+1)$ for $1 <= i < n$. They are known for their mysteriously simple generating function and their relationship to Euler's odd-distinct partition theorem. In this talk, we generalize to s-lecture hall partitions, satisfying $x_i/s_i <= x_{i+1}/s_{i+1}$ for $1 <= i < n$, where s is an arbitrary positive integer sequence. We introduce s-inversion sequences and show that the enumeration of s-lecture hall partitions gives rise to new families of Eulerian polynomials, defined by ascent statistics on s-inversion sequences. These results unify, generalize, and extend several familiar results about Eulerian polynomials and lecture hall partitions. This includes joint work with M. Schuster, G. Viswanathan, and T. Pensyl.

AUTHOR: Jeffrey Shallit
AFFILIATION: University of Waterloo, Canada
TITLE: 50 Years of Fine and Wilf
ABSTRACT: It is now almost 50 years since the classic paper of Nathan Fine and Herb Wilf entitled "Uniqueness theorems for periodic functions". That paper - their only joint one - led to many other interesting results in combinatorics on words and other areas. In this talk I will give a new proof of the Fine-Wilf theorem and discuss some of the progress in the last 50 years.

AUTHOR: Richard Stanley
AFFILIATION: MIT, USA
TITLE: Products of Cycles
ABSTRACT: We will discuss two aspects of multiplying an n-cycle by another permutation in the symmetric group S_n of all permutations of $1, 2, ..., n$. (1) It is easy to see that if $n > 1$, then the probability that 1 and 2 are in the same cycle of a random permutation in S_n is $1/2$. Miklos Bona conjectured that if n is also odd, then the probability that 1 and 2 are in the same cycle of a product of two random n-cycles in S_n is $1/2$. We will explain a proof of this conjecture and of many extensions of it. For instance, if $n > k - 1$ and $n - k$ is odd, then the probability that $1, 2, ..., k$ are in k different cycles of a product

of two random n-cycles is $1/k!$. Many open problems and conjectures remain. Much of this work was done in collaboration with Rosena R. X. Du. (2) We consider the distribution of the number of cycles in the product of an n-cycle with a permutation of fixed cycle type. This distribution was first obtained by Zagier and has connections with such topics as Riemann surfaces, polynomials with real zeros, graph theory, and Kerov's character polynomials. In particular we show that the zeros of the generating function for this distribution all have real part 0.

AUTHOR: John Stembridge
AFFILIATION: University of Michigan, USA
TITLE: A Finitness Theorem for W-graphs
ABSTRACT: Consider an integer polynomial such as $p(t) = t(t+1)(t-1)(t-3)$. Can you prove that there are only finitely many square, nonnegative integer matrices A that are "irreducible" (i.e., cannot be permuted into block triangular form) and satisfy $p(A) = 0$? Can you generate them all? Questions like these are the tip of an iceberg that has implications for W-graphs, a category of edge-weighted graphs that encode certain representations of a Weyl group W (or its Hecke algebra). In particular, we can show that for any given Weyl group W, there are only finitely many strongly connected W-graphs with nonnegative integer edge weights. Every Kazhdan-Lusztig W-cell provides an example of such a graph, and in the symmetric groups $W = S_n$ for $n \leq 10$ they are (modulo a few mild technical hypotheses) the only examples.

AUTHOR: Volker Strehl
AFFILIATION: Universitaet Erlangen, Germany
TITLE: Aspects of a Combinatorial Annihilation Process
ABSTRACT: The asymmetric annihilation process has been introduced by A. Ayyer and K. Mallick (J Phys A, 2010) and has been further studied by A. Ayyer and the author (FPSAC 2010). It is similar in spirit to the familiar TASEP model, but it also allows for the annihilation of neighbouring particles. In this talk I will describe an algebraic framework for a fully parametrized version of the annihilation process, including the derivation of its partition function.

AUTHOR: Michelle Wachs
AFFILIATION: University of Miami, USA
TITLE: Unimodality of q-Eulerian Numbers and p,q-Eulerian Numbers
ABSTRACT: The Eulerian numbers enumerate permutations in the symmetric group S_n by their number of excedances or by their number of descents. It is well known that they form a symmetric and unimodal sequence of integers. In this talk, which is based on work with John Shareshian and Anthony Henderson, we consider the q-analog of the Eulerian numbers obtained from the joint distribution of the major index and the excedance number, and the p,q-analog of the Eulerian numbers obtained by considering the joint distribution of the major index, descent number and excedance number. We show that the q-Eulerian numbers form a symmetric and unimodal sequence of polynomials in q and the p,q-Eulerian numbers refined by cycle type form a symmetric and unimodal sequence of polynomials in p and q. The proofs of these results rely on the Eulerian quasisymmetric functions introduced in joint work with Shareshian, on symmetric and quasisymmetric function theory, and on representation theory of the symmetric group.

Author: Herbert S. Wilf
Affiliation: University of Pennsylvania, USA
Title: Two problems in combinatorial biology
Abstract: The talk concerns two problems that were studied by Warren Ewens and myself and published in PNAS. The first concerns recognition of a disease epidemic as opposed to just a coincidental large number of cases in a small geographical area. The second is about evolution. One objection that has been raised to Darwinian evolution is that too much time would be required for the necessary numbers of random mutations to happen, as needed to encode a complex organism. We show that if a simple model of the way natural selection and evolution work together is studied, it shows that in a very short time, a simple word can be transformed into a complex one with random transformations of the letters. So in fact there is plenty of time for evolution.

Author: Doron Zeilberger
Affiliation: Rutgers University, USA
Title: Automatic Generation of Theorems and Proofs on Enumerating Consecutive WILF-classes
Abstract: Shalosh B. Ekhad can ANSWER (in WILF's sense of the word) the question "how many permutations avoid such and such (classical) pattern" for quite a few "such and such" (classical) patterns, but, so far, and most probably never, for ALL patterns. But it sure can ANSWER (in W's sense) the analogous question for consecutive-WILF patterns, introduced by Sergi Elizalde and Marc Noy. Joint work with Andrew Baxter, Brian Nakamura, and of course, Shalosh B. Ekhad.

Author: Eugene Zima
Affiliation: Wilfrid Laurier University, Canada
Title: Synthetic division in the context of indefinite summation
Abstract: A modification of an algorithm for indefinite rational summation is presented. It is based on direct divisibility test in the ring of linear difference operators with polynomial coefficients. When the rational function is not summable it provides solution to the additive decomposition problem with minimal degree of the denominator of the rational part. The algorithm solves the problem in time which is polynomial in the size of the input and linear in the minimal possible size of the output. This removes all non-essential dependencies of the running time of the algorithm from the dispersion of the input. The result is extended to the case of quasi-rational indefinite summation. Prototype implementation of the algorithm in Maple together with succinct representation of the intermediate results is discussed.
