The effect of corrected gravitational acceleration in the depth and diameter in tsunamis of meteoric origin

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Abstract

Among nature's cosmic catastrophes can be the ones caused by a falling meteorites/asteroids of considerable sizes. In this presentation we examine two different of meteorite sizes falling at the ocean. Thus, we study the effect that the Earth's corrected gravity for geocentric latitude, J_2 harmonic (oblatness) and rotation has upon impact in the depth of a created crater, as well a in its radius. We have found that that the total correction effect of geocentric latitude, oblateness,and rotation results to a 0.30% increase in the depth of the crater if the impact occurs at the equator when compared to a fall at the poles. Similarly the radius increases by 0.12% if the impact occurs at the equator. We conclude that meteoric/asteroid impacts at the equator will results to more catastrophic results and higher tsunami heights when compares to those of polar impacts.

Introduction

Progress in celestial mechanics and solar system dynamics in the twentieth and twenty-first centuries resulted in the extensive study of various solar system meteorite bodies, so that their orbits and possible scenario of meteorite collision course could be determined. Furthermore, accumulation of solar system knowledge coming from various missions played an important role in determining the possibility of a possible meteoric collision with the Earth.

We now know that meteoric sizes vary from tens of meter to kilometers. A considerable number of these bodies approaches the Earth to very close distances, some of which are comparable to the Earth - Moon distance. Collision of large bodies with the Earth is truly a rare event, where collision with smaller bodies as well as medium sized bodies, have happened quite often in the past. Earth has often collided with meteoritic bodies something that expected to double every 5.8 years. So far we have knowledge of approximately two hundred craters which due to data inconsistency can probably double Kozelknov et al. (2015).

Asteroid and comet impacts have also played a major role in the biological and geological history of the Earth. In particular we say that two thirds of the space objects that fall on the Earth end up impacting the water, thus the possibility of a tsunami creation that can be propagated in great distances, is a very probable event. These are the tsunamis that are created due to the collapse of pelagic impact cavities (Hills at al. 1994), and (Nemtchinov et al. 1997). Finally, we say that our planet exhibits evidence of asteroid impacts and impact tsunamis as well. As an example we mention that Gersonde et al. (1997) have documented the collision of a 1km or probably larger in diameter in the asteroid in the south ocean during the late Pliocene (2.15 Ma) . In this paper we examine the effect that corrected gravitational acceleration has upon impact in the meteorite crater depth as well as upon it's the crater radius , for two meteorites of 50 and 100 m respectively.

Asteroid impact tsunami theory basics and initial cavity shape specifics modeling method

The tsunami energy increases when water mass is deposited on the lids of the created cavity, which at the same time increases the energy of the cavity. The additional energy is composed of shorter length waves than the cavity's diameter, and it results in slow travelling that does not increase the tsunami's peak amplitude. Following (Ward and Asphaug, 2000), and for initial transient cavities i.e. cavities where all the water is deposited into the lip, would imply that and therefore there is no net water loss, and therefore we can write that the depth of the cavity is given by:

$$D_c = \left(\frac{2\varepsilon\rho_i R_i^3 V_i^2}{\rho_w g R_c^2}\right)^{1/2}$$

furthermore, assuming a general relationship between cavity and radius to be (Ward and Asphaug, 2000) . The radius of the cavity is given by the equation:

$$R_c = R_i\left[\left(\frac{2\varepsilon V_i^2}{gR_i}\right)^{1/4}\left(\frac{\rho_i}{\rho_w}\right)^{1/4}\left(\frac{\rho_i}{\rho_w}\right)^{\frac{1}{4}}\left(\frac{1}{q\alpha^{\ldots}}\right)^{\beta}\right]$$

The gravitational acceleration on the surface of the Earth and at orbital point

In our effort to study the effect of gravity on the depth and radius of the cavity if any, let us consider a corrected acceleration of gravity *g*. This would imply that we will take into account the dependence of gravitational acceleration a) on geocentric latitude and b) on the Earth's oblateness that is expresses via the J_2 harmonic coefficient, and finally the rotation of the Earth Following Haranas, et al., (2012, 2015) we write the acceleration of gravity as the sum of three different components namely:

$$g(\phi) = \frac{GM_E}{R_{eq}^2\left(1-f'\sin^2\phi\right)^2} - \frac{3GM_E J_2\left(3\sin^2\phi-1\right)}{2R_{eq}^2\left(1-f'\sin^2\phi\right)^4} - R_{eq}\omega_E^2\left(1-f'\sin^2\phi\right)\cos^2\phi$$

where is the radial distance from the center of the Earth to an external surface point, M_E is the mass of the Earth, R_E is the radius of the Earth, J_2 is the zonal harmonic coefficient that describes the oblateness of the Earth, the angular velocity of the Earth, and ϕ_E the geocentric latitude of the meteorite falling site. Zonal harmonics are simply bands of latitude, whose boundaries are the roots of a Legendre polynomial. Spherical harmonics in general are very important concept in solar system research and in particular in the modelling of planetary gravity fields. For example in Hadjifotinou (2000) the author uses a gravitational potential that includes a J_2 as well as a J_4 harmonics in predicting numerically the motion of Saturn's satellites. Similarly, in Iorio (2011) the author derives the precession of the ascending node of a satellite due to Lense-Thirring effect as a function of the J_2 harmonic. This particular gravitational harmonic coefficient is a result of the Earth's shape and is about 1000 times larger than the next harmonic coefficient J_3 and its value is equal to $J_2 = 0.0010826269$ (Vallado and McCain, 2000).

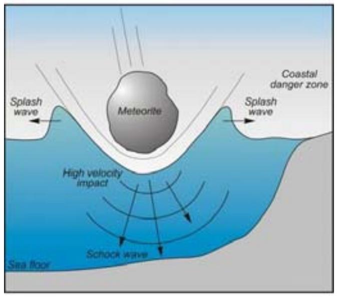

Figure 1 Meteorite falling in the ocean and creating a crater of depth *D* and radius *R*, that eventually starts the tsunami activity.

Numerical Results

To proceed with our numerical calculation let us assume the following values for the parameters entering our calculations. $M_E = 5.9742 \times 10^{24}$ kg is the mass of the Earth, and $R_E = 6378.1363$ km is the radius of the Earth, $J_2 = 0.0010826269$ (Vallado and Mac Cain, 2007), $f = 0.003353813178$ is the flattening coefficient of the Earth, $\omega_E = 7.292115\times 10^{-5}$ rad/s is the rotational velocity of the Earth (ibid, 2007). Furthermore, we assume meteorites with radii $R_i = 50$, 100, 500 m respectively, and $q = 0.10$, 0.083, 0.054 (Ward and Asphaug, 2000) of density $\rho_i = 3000$ kg/m³, and a Schmidt-Holsapple parameter $\beta = 0.22$ (Ward and Asphaug, 2000), which has a physical connection to the aspect ratio of a crater and if the impact energy conversion factor holds rather constant at approximately $\approx$ 15%, finally from laboratory experiments in water $\alpha = 1.27$ (Ward and Asphaug, 2000).

First we calculate the depth of cavity upon impact with constant acceleration of gravity $g = 9.81$ m/s², for two meteorites with radius $R_i = 50$ and 100 m respectively and with corresponding q's = 0.10 and 0.083 (Ward and Asphaug, 2000), density $\rho_i = 3000$ kg/m³, $V_i = 20$ km/s, ε =16%. We have assumed that the meteorites fall in the ocean with density ρ_w= 1027 kg/m³ and therefore we obtain that the cavity's corresponding depths are $D_c = 1292.32$ m and 2128.86 m respectively. These numbers are in very close agreement with the results given in (Ward and Asphaug, 2000). Similarly, taking into account the corrected gravitational acceleration i.e. J_2 and rotation of the Earth we find the results tabulated in table 1 below using the modified gravity corrected formula given above the tablre:

$$D_c = \left(\frac{2\varepsilon\rho_i R_i^3 V_i^2}{\rho_w R_c^2\left(\frac{GM_E}{R_{eq}^2\left(1-f'\sin^2\phi\right)^2} - \frac{3GM_E J_2\left(3\sin^2\phi-1\right)}{2R_{eq}^2\left(1-f'\sin^2\phi\right)^4} - R_{eq}\omega_E^2\left(1-f'\sin^2\phi\right)\cos^2\phi\right)}\right)^{1/2}$$

Table 1 Corrected gravity cavity depth effect for a 50 m and 100 diameter meteorite at various geocentric latitudes

Geocentric Latitude ϕ [deg]	Uncorrected Gravity Cavity Depth Calculation *D* [m]	Uncorrected Gravity Cavity Depth Calculation *D* [m]	Corrected Gravity Cavity Depth Calculation *D* [m]	Corrected Gravity Cavity Depth Calculation *D* [m]
	Meteorite Diameter 50 m	Meteorite Diameter 100 m	Meteorite Diameter 50 m	Meteorite Diameter 100 m
0	1292.32	2128..86	1294.48	2132.42
30			1293.62	2131.00
45			1292.76	2129.59
60			1291.91	2128.19
90			1291.07	2126.79

Table 2 Corrected gravity cavity radius effect for a 50 m and 100 diameter meteorite at various geocentric latitudes

Geocentric Latitude ϕ [deg]	Uncorrected Gravity Cavity Radius Calculation *R* [m]	Uncorrected Gravity Cavity Radius Calculation *R* [m]	Corrected Gravity Cavity Radius Calculation *R* [m]	Corrected Gravity Cavity Radius Calculation *R* [m]
	Meteorite Diameter 50 m	Meteorite Diameter 100 m	Meteorite Diameter 50 m	Meteorite Diameter 100 m
0	1692.29	2908.84	1693.54	2910.99
30			1693.04	2910.13
45			1692.55	2909.28
60			1692.06	2908.44
90			1691.57	2907.60

Conclusions

Correcting the Earth's gravitational acceleration for the latitude dependence, oblateness and rotation we have found that upon impact a falling into the ocean meteorite results to a crater depth and radius that is higher at the equator and smaller at the poles . The % difference for meteorite considered between the depth at the equator and that at the poles is approximately 0.30% where between the radius for the same geocentric locations is about 0.12%. This would imply, that a meteorite/asteroid if fell at the equator given its size and velocity would create a grater depth and diameter crater which potentially will result in a higher maximum tsunami height.

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