

# Van der Waerden/Schrijver-Valiant like Conjectures and Stable (aka Hyperbolic) Homogeneous Polynomials : One Theorem for all .

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## Abstract

Let  $p(x_1, \dots, x_n) = p(X)$ ,  $X \in R^n$  be a homogeneous polynomial of degree  $n$  in  $n$  variables. Such polynomial  $p$  is called **H-Stable** if  $p(z_1, \dots, z_n) \neq 0$  provided the real parts  $Re(z_i) > 0$ ,  $1 \leq i \leq n$ . This notion from *Control Theory* is closely related to the notion of *Hyperbolicity* intensively used in the *PDE* theory.

The main theorem, presented in this talk, states that if  $p(x_1, \dots, x_n)$  is a homogeneous **H-Stable** polynomial of degree  $n$ ,  $deg_p(i)$  is the maximum degree of the variable  $x_i$ ,  $C_i = \min(deg_p(i), i)$  and  $p(x_1, x_2, \dots, x_n) \geq \prod_{1 \leq i \leq n} x_i$ ;  $x_i > 0$ ,  $1 \leq i \leq n$  then the following inequality holds

$$\frac{\partial^n}{\partial x_1 \dots \partial x_n} p(0, \dots, 0) \geq \prod_{1 \leq i \leq n} \left( \frac{C_i - 1}{C_i} \right)^{C_i - 1} \geq \frac{n!}{n^n} \quad (1)$$

This inequality is a vast (and unifying) generalization of the van der Waerden conjecture on the permanents of doubly stochastic matrices as well as the Schrijver-Valiant conjecture on the number of perfect matchings in  $k$ -regular bipartite graphs. These two famous results correspond to the **H-Stable** polynomials which are products of linear forms.

The main goal of the talk is to give a reasonably complete proof of inequality (1). Our proof is relatively simple and "noncomputational"; it actually slightly improves Schrijver's lower bound, and uses very basic properties of complex numbers and AM/GM inequality.

Undergraduate and graduate students are encouraged to attend.

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