

2-Designs admitting a flag-transitive automorphism group

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Abstract

A $2-(v, k, \lambda)$ design $\mathcal{D} = (\mathcal{P}, \mathcal{B})$ consists of a set \mathcal{P} of v points and a set \mathcal{B} of blocks such that each block is a k -subset of \mathcal{P} and each pair of distinct points is contained in exactly λ blocks. A flag of \mathcal{D} is an incident point-block pair, and a group G of automorphisms of \mathcal{D} is *flag-transitive* if it acts transitively on the set of flags. Such a group G is transitive on both \mathcal{P} and \mathcal{B} . Also, G is said to be *point-imprimitive* if it leaves invariant a partition of the point set \mathcal{P} in classes of size c with $1 < c < v$, and *point-primitive* otherwise.

If $\lambda = 1$, then G acts point-primitively on \mathcal{D} by a celebrated result of Higman and McLaughlin [5], and a classification of such 2-designs was achieved by Buekenhout et al. in [4] except when v is a power of a prime and $G \leq A\Gamma L_1(v)$.

If $\lambda > 1$, there are many known families of flag-transitive point-imprimitive 2-designs. Recently, as an effort of several authors [1, 2, 3, 6, 7, 8, 9], a classification of flag-transitive 2-designs with $\lambda = 2$ has been achieved except when v is a power of a prime and $G \leq A\Gamma L_1(v)$.

In my talk, I will give an overview on flag-transitive 2-designs, both in the primitive and imprimitive case, present some constructions, and provide some recent classification results.

The talk is based on joint works with S. H. Alavi, M. Bayat, A. Daneshkhakh, H. Liang, C. E. Praeger, Y. Zhao, Z. Zhang and S. Zhou.

Keywords: Flag-transitive designs, 2-designs, permutation groups.

MSC: 05B05, 20B25, 05E16

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