

# Near-factorization of finite groups

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## Abstract

Let  $(G, \cdot)$  be a finite multiplicative group with identity  $e$ . For  $A, B \subseteq G$ , define

$$AB = \{gh : g \in A, h \in B\}$$

and note that  $AB$  is a multi-set.

We say that  $(A, B)$  is a *near-factorization of  $G$  with index  $\lambda$*  if  $|A| \times |B| = \lambda(|G| - 1)$  and each element of  $G \setminus \{e\}$  occurs  $\lambda$  times in the product  $AB$ . We abbreviate this by writing  $AB = \lambda(G \setminus \{1\})$ . If  $(A, B)$  is a near-factorization with index  $\lambda$ , then we say that  $B$  is a  $\lambda$ -mate of  $A$ . A  $\lambda$ -mate with  $\lambda = 1$  is simply called a mate.

Some new structural properties of near-factorizations in certain classes of groups are established. In particular if there is a near-factorization  $(A, B)$ , then there is an explicate formula for  $B$  in terms of  $A$ . This leads to an efficient method for computing the  $\lambda$ -mate  $B$  of a subset  $A \subseteq G$ , if it exists.

All noncyclic abelian groups of order less than 200 were examined in a search for a possible nontrivial near-factorization with index 1 and all of these possibilities were ruled out, either by theoretical criteria or by exhaustive computer searches. (In contrast, index 1 near-factorizations in cyclic or dihedral groups are known to exist by previous results.)

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