FIXED POINT FREE AUTOMORPHISMS

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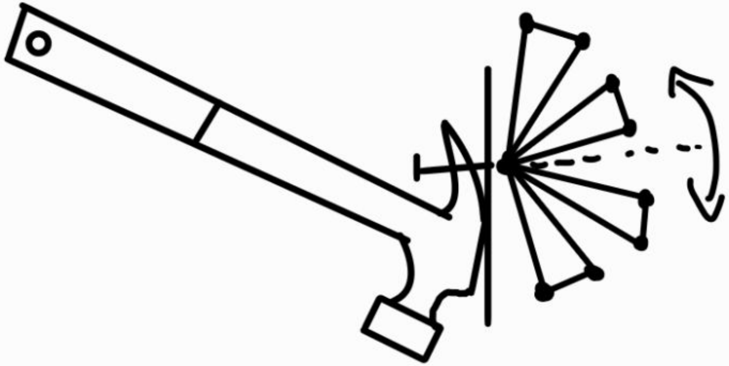
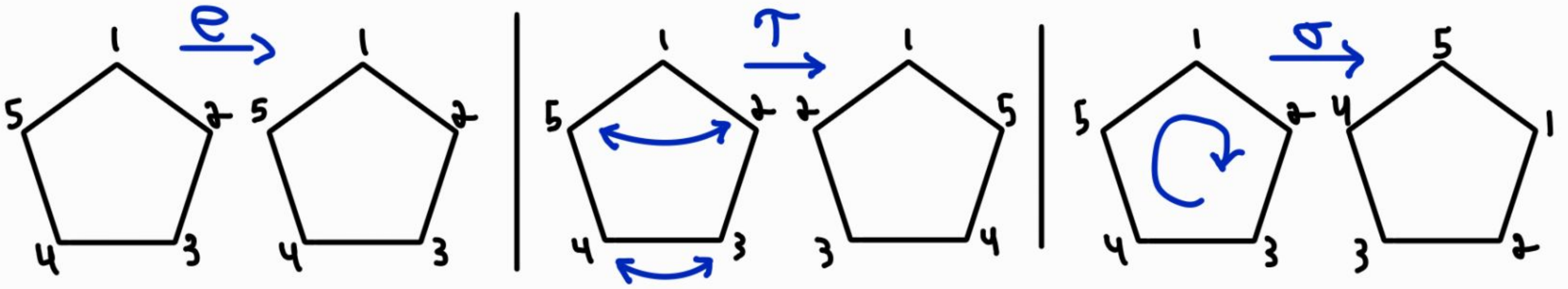
Joint work w/ Abiad, Coutinho, dos Santos, and Zeijlemaker



5th Pythagorean Conference

- An **Automorphism** of G is a bijection

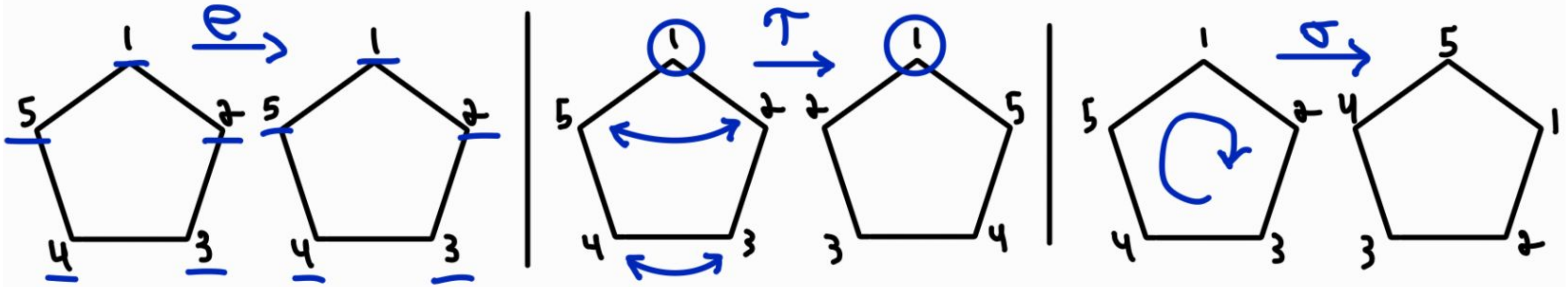
$\phi: V(G) \to V(G)$ s.t. $uv \in E(G) \iff \phi(u)\phi(v) \in E(G)$.



- **Aut(G)** = set of automorphisms of G.

Thm. Frucht '38) Every group is the automorphism group of some graph.

- $\phi \in Aut(G)$ is **fixed point free (FPF)** if
$\forall v \in V(G)$, $\phi(v) \neq v$.



FPFAut
Input: Graph G
Question: $\exists \phi \in Aut(G)$: ϕ is FPF?

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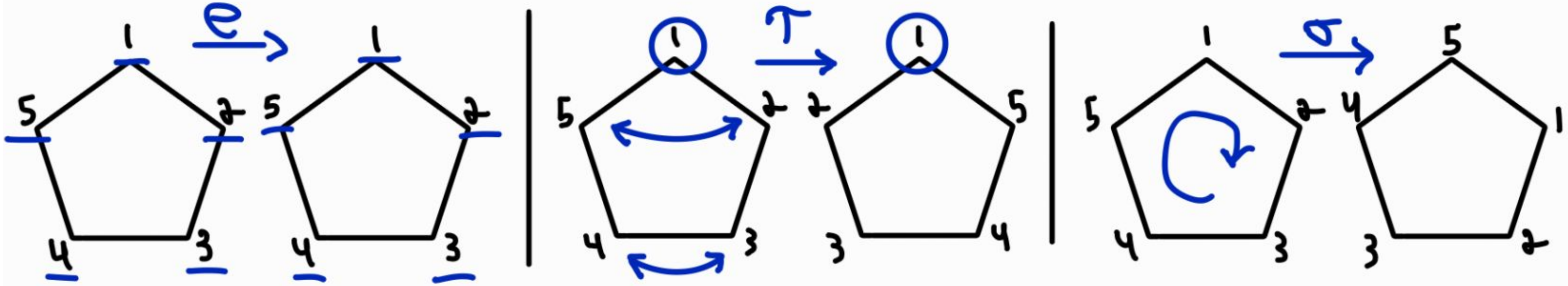

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Thm Lubiw '81) FPFAut is NP-H.

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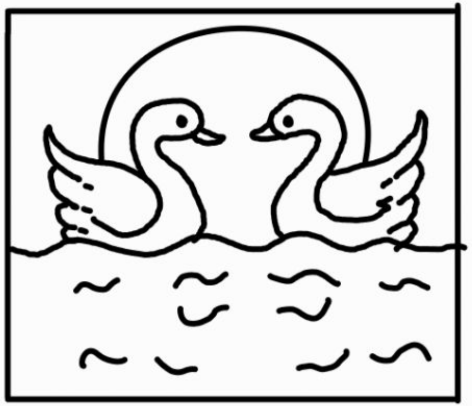
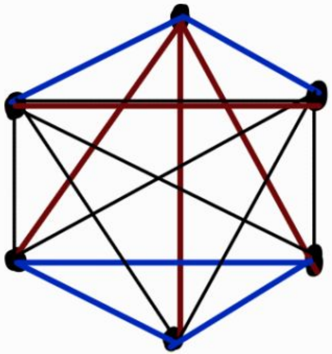
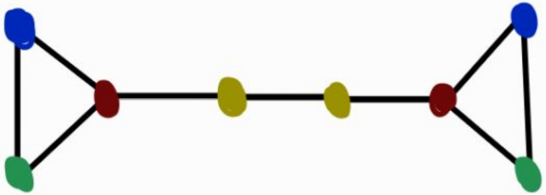
Graph Drawing



Combinatorial Games



Equitable Partitions



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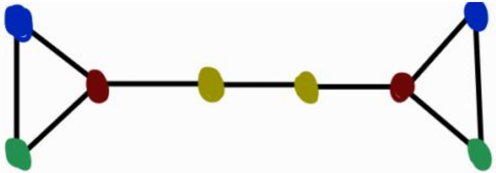
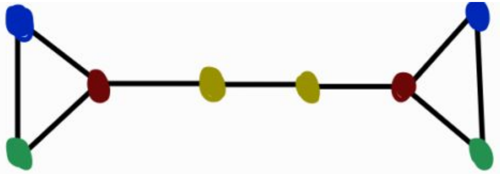


- A partition $\mathcal{P} = \{P_1, \dots, P_m\}$ of $V(G)$ is a 2-homogeneous equitable partition if

$|P_i| = 2$ and if $P_i = \{v_1, v_2\} \Rightarrow d(v_1, P_j) = d(v_2, P_j) \ \forall j$

Equitable Partitions



- A partition $\mathcal{P}=\{P_1,\dots,P_m\}$ of $V(G)$ is a 2-homogeneous equitable partition if

$|P_i|=2$ and if $P_i=\{v_1,v_2\} \Rightarrow d(v_1,P_j)=d(v_2,P_j)\ \forall j$

Thm Abiad, Hojny, Zeijlemaker '22)
Decide if a graph G has a 2-hom. eq. part. is NP-c.

Thm Abiad, Hojny, Zeijlemaker '22)
Decide if a cograph G has a 2-hom. eq. part. can be solved in time $|V(G)|^{O(1)}$.

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. Let Γ be a group acting on a set X.

Burnside Lemma) $\#\text{orbits of } X = \frac{1}{|\Gamma|} \sum_{g \in \Gamma} \text{fix}(g)$

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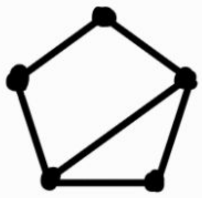
$$1 = \frac{1}{|\mathrm{Aut}(G)|}\left[|V(G)| + \sum_{\substack{\phi \in \mathrm{Aut}(G) \\ \phi \neq e}} \mathrm{Fix}(\phi) \right]$$

- Let $H \leqslant G$ denote that $H \cong G[S]$ for some $S \subseteq V(G)$.

- G is H-free if $H \nleqslant G$.



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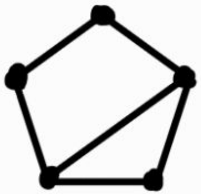

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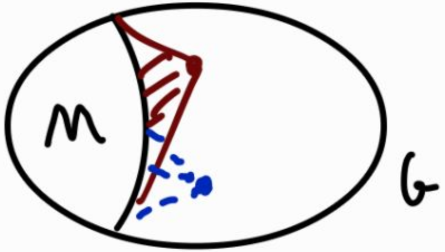
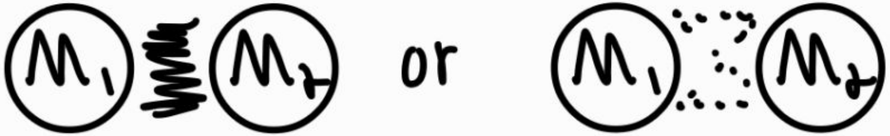
Thm) FPFAut is NP-H for H-free graphs, unless $H \leqslant P_4$.

- Module : $M \subseteq V(G)$: $\forall v \in V(G) \setminus M$

$N(v) \cap M = M$ or $N(v) \cap M = \emptyset$

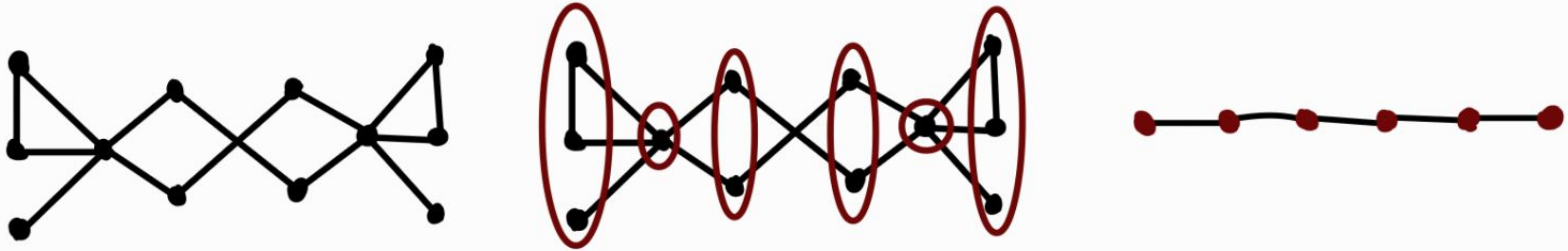


- M_1, M_2 disjoint modules. Then, either



- A partition $P = \{M_1, \ldots, M_m\}$ of $V(G)$ is **modular** if M_i is a module $\forall i$.

- Quotient graph G/P : $V(G/P) = P$, $M_i \sim M_j$ if $E(M_i, M_j) \neq \emptyset$



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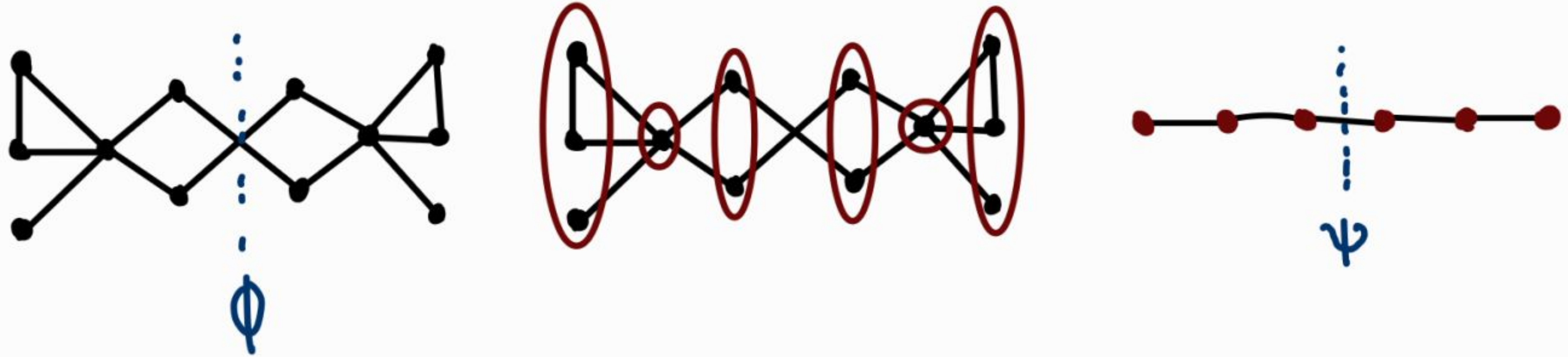

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Graph Class	Quotient Graph
P_4-free (cographs) [S'73]	Complete 

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Thm) FPFAut can be solved in time $|V(G)|^{O(1)}$ if G is cograph, bounded m.w. graph

Graph Class	Quotient Graph
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Bounded modular width [GLO'13]	, "Small" 

Thm) FPFAut can be solved in time $|V(G)|^{O(1)}$ if G is cograph, bounded m.w. graph, tree-cograph

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P_4-free (cographs) [S'73]	Complete 
Bounded modular width [GLO'13]	 , "Small" 
Tree-cograph [T'88]	 , Tree 

Thm) FPFAut can be solved in time $|V(G)|^{O(1)}$ if G is cograph, bounded m.w. graph, tree-cograph, P_4-sparse.

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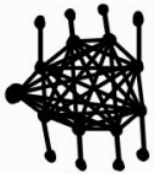
Bounded modular width [GLO'13]	, "Small" 
Tree-cograph [T'88]	, Tree 
P_4-sparse [JO'97]	, Spider 

THANK YOU!

(Σας ευχαριστώ)