One-Weight Sum-Rank Metric Codes

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Motivation

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Aim

Our goal is to classify one-weight codes in the sum-rank setting, extending foundational results like Bonisoli's theorem.

Outline

1. Sum-Rank Metric Codes
2. Geometry of Sum-Rank Metric Codes
3. One-Weight Codes (Constant Rank List/Profile)

Sum-Rank Metric Codes

$$\mathbf{n} = (n_1, \ldots, n_t) \in \mathbb{N}^t, \quad n_1 \geqslant \cdots \geqslant n_t, \quad N = n_1 + \cdots + n_t$$

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$$v = (v_1, \dots, v_t) \in \mathbb{F}_{q^m}^{\mathbf{n}}, \quad v_i \in \mathbb{F}_{q^m}^{n_i}$$

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$$v = (v_1, \ldots, v_t) \in \mathbb{F}_{q^m}^{\mathbf{n}}, \quad v_i \in \mathbb{F}_{q^m}^{n_i}$$

$$w = (w_1, \ldots, w_{n_i}) \in \mathbb{F}_{q^m}^{n_i}, \quad \mathrm{rk}_{\mathbb{F}_q}(w) = \dim_{\mathbb{F}_q}(\langle w_1, \ldots, w_{n_i} \rangle_{\mathbb{F}_q})$$

Sum-Rank Metric Codes

$$x = (x_1, \ldots, x_t), \quad y = (y_1, \ldots, y_t) \in \mathbb{F}_{q^m}^{\mathbf{n}}$$

$$d(x, y) = w(x - y) = \sum_{i=1}^{t} rk_{\mathbb{F}_q}(x_i - y_i)$$

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If $\mathcal{C} \leqslant \mathbb{F}_{q^m}^{\mathbf{n}} \implies d(\mathcal{C}) = \min\{d(x, y) : x, y \in \mathcal{C}, x \neq y\}$

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If $\mathcal{C} \leqslant \mathbb{F}_{q^m}^{\mathbf{n}} \implies [\mathbf{n}, k, d]_{q^m/q}$**code**

$$\mathbb{F}_{q^m}^{\mathbf{n}} = \mathbb{F}_{q^m}^{1} \oplus \cdots \oplus \mathbb{F}_{q^m}^{1}, \text{ here } \mathbf{n} = (1, \ldots, 1)$$

Hamming Metric Codes

If $\mathcal{C} \leqslant \mathbb{F}_{q^m}^{\mathbf{n}} \implies [\mathbf{n}, k, d]_{q^m/q}$ code

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Hamming Metric Codes

If $\mathcal{C} \leqslant \mathbb{F}_{q^m}^{\mathbf{n}} \implies [\mathbf{n}, k, d]_{q^m/q}$ code

$$\mathbb{F}_{q^m}^{\mathbf{n}} = \mathbb{F}_{q^m}^{n_1} \text{ here } t = 1 \text{ and } \mathbf{n} = n_1$$

Rank Metric Codes

If $\mathcal{C} \leqslant \mathbb{F}_{q^m}^{\mathbf{n}} \implies [\mathbf{n}, k, d]_{q^m/q}$ code

Theorem (Martínez-Peñas (2020) Alfarano, Lobillo, Neri, and Wachter-Zeh (2021))

The group of $\mathbb{F}_{q^m}$-linear isometries of the space $(\mathbb{F}_{q^m}^{\mathbf{n}}, \mathrm{d})$ is

$$((\mathbb{F}_{q^m}^*)^t \times \mathrm{GL}(\mathbf{n}, \mathbb{F}_q)) \rtimes \mathcal{S}_{\lambda(\mathbf{n})},$$

which (right)-acts as $(x_1, \ldots, x_t) \cdot (\mathbf{a}, A_1, \ldots, A_t, \pi) \longmapsto (a_1 x_{\pi(1)} A_1 \mid \ldots \mid a_t x_{\pi(t)} A_t)$.

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Definition

Two $[\mathbf{n}, k]_{q^m/q}$ sum-rank metric codes $\mathcal{C}_1, \mathcal{C}_2$ are **equivalent** if there is an $\mathbb{F}_{q^m}$-linear isometry ϕ, such that $\phi(\mathcal{C}_1) = \mathcal{C}_2$.

Geometry of Sum-Rank Metric Codes

Generator matrix $G = (G_1| \ldots |G_t) \in \mathbb{F}_q^{k \times N}, N = n_1 + \cdots + n_t$

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Definition

$[\mathbf{n}, k, d]_{q^m/q}$ code is **non-degenerate** if any generator matrix $G \in \mathbb{F}_q^{k \times N}$ of $\mathcal{C}$ has a property $\dim_{\mathbb{F}_q}(\mathrm{colsp}_{\mathbb{F}_q}(G)) = N$

$G = (G_1 \mid \dots \mid G_t)$, where $G_i \in \mathbb{F}_{q^m}^{k \times n_i}$. The $\mathbb{F}_q$-span of the columns of G_i and the maps

$$\begin{aligned} \psi_{G_i} : \quad \mathbb{F}_q^{n_i} &\longrightarrow \mathcal{U}_i \\ \lambda &\longmapsto \lambda G_i^\top \end{aligned}$$

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$$[\mathbf{n}, k]_{q^m/q} \text{ code } \iff (\mathcal{U}_1, \ldots, \mathcal{U}_t) \leqslant_{\mathbb{F}_q} \mathbb{F}_{q^m}^k, \text{ where } \mathcal{U}_i \leqslant_{\mathbb{F}_q} \mathbb{F}_{q^m}^k$$

$$\mathbf{n} = (\dim_{\mathbb{F}_q}(\mathcal{U}_1), \ldots, \dim_{\mathbb{F}_q}(\mathcal{U}_t))$$

Theorem (Neri, Santonastaso and Zullo (2023))

$\mathcal{C}$ be a non-degenerate $[\mathbf{n}, k, d]_{q^m/q}$ code, $\mathcal{U}_i$ be the $\mathbb{F}_q$-span of the columns of G_i, for $v \in \mathbb{F}_{q^m}^k$

$$\mathrm{w}(vG) = N - \sum_{i=1}^{t} \dim_{\mathbb{F}_q}(\mathcal{U}_i \cap v^{\perp})$$

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$$(\mathcal{U}_1, \ldots, \mathcal{U}_t) \leqslant_q \mathbb{F}_{q^m}^k, \text{ where } \mathcal{U}_i \leqslant_{\mathbb{F}_q} \mathbb{F}_{q^m}^k$$

$$n_i = \dim_{\mathbb{F}_q}(\mathcal{U}_i)$$

$[\mathbf{n}, k, d]_{q^m/q}$**-system**

$$d = N - \max\left\{\sum_{i=1}^{t} \dim_{\mathbb{F}_q}(\mathcal{U}_i \cap v^{\perp}) : v \in \mathbb{F}_{q^m}^k\right\}$$

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Definition

$(\mathcal{U}_1, \ldots, \mathcal{U}_t)$,$(\mathcal{V}_1, \ldots, \mathcal{V}_t)$ systems are **equivalent** if there exists an isomorphism $\varphi \in \mathrm{GL}(k, \mathbb{F}_{q^m})$, an element $\mathbf{a} = (a_1, \ldots, a_t) \in (\mathbb{F}^*_{q^m})^t$,$\sigma \in \mathcal{S}_t$ such that

$$\varphi(\mathcal{U}_i) = a_i \mathcal{V}_{\sigma(i)}.$$

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$\mathfrak{C}[\mathrm{n}, k, d]_{q^m/q}$ = Equivalence classes of $[\mathrm{n}, k, d]_{q^m/q}$-codes

$\mathfrak{U}[\mathrm{n}, k, d]_{q^m/q}$ = Equivalence classes of $[\mathrm{n}, k, d]_{q^m/q}$-systems

Theorem (Neri, Santonastaso and Zullo (2023))

There is a one-to-one correspondence between $\mathfrak{C}[\mathbf{n}, k, d]_{q^m/q}$ and $\mathfrak{U}[\mathbf{n}, k, d]_{q^m/q}$

One-Weight Codes (Constant Rank List/Profile)

One-Weight Codes

$$\mathcal{C} \leqslant \mathbb{F}_{q^m}^{\mathbf{n}}$$

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Theorem (Bonisoli (1983))

One-weight Hamming metric code $\iff$ simplex Hamming metric code.

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Theorem (Bonisoli (1983))

One-weight Hamming metric code $\iff$ simplex Hamming metric code.

Theorem (Randrianarisoa, Alfrano, Borello, Neri, and Ravagnani (2020))

One-weight rank-metric code $\iff$ simplex rank-metric code

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Neri, Santonastaso and Zullo (2023)

- Definition of Simplex sum-rank metric code.
- They showed that Simplex sum-rank metric codes are not the only one weight code.

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Attention

A Bonisoli-like result in the sum-rank metric cannot hold.

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Question

Can we add assumptions to one-weight codes to get a full classification?

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Can we get new constructions of one weight codes?

Constant Rank List

Definition

$c \in \mathcal{C}$, the **Rank list**

$$\rho(c) = (w(c_1), \dots, w(c_t))$$

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$$\tau : \mathbb{N}^t \to \mathbb{N}^t$$

for any $(a_1, a_2, \ldots, a_t) \in \mathbb{N}^t$, we have

$$\tau(a_1, \ldots, a_t) = (b_1, \ldots, b_t)$$

$$b_1 \geqslant b_2 \geqslant \cdots \geqslant b_t,$$

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Definition

The **Rank profile** $\mu(X)$ is rearranging $\rho(X)$ entries in non-increasing order

Constant Rank List

Definition

Let $\mathcal{C} \leqslant \mathbb{F}_{q^m}^n$, $(\rho_1, \ldots, \rho_t) \in \mathbb{N}^t$. If for any $c \in \mathcal{C} \setminus \{0\}$ we have $\rho(c) = (\rho_1, \ldots, \rho_t) \implies \mathcal{C}$ is a **constant rank list** $(\rho_1, \ldots, \rho_t)$.

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Notation

Let $c = (c_1, \ldots, c_t) \in \mathbb{F}_{q^m}^{\mathbf{n}}$ and let $i \in [t]$, the i-th component of c, denoted as $c_{|i}$, is defined as

$$c_{|i} = c_i \in \mathbb{F}_{q^m}^{n_i}.$$

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***i*-th projection** of $\mathcal{C}$ as

$$\mathcal{C}_i = \{c_{|i} : c \in \mathcal{C}\} \subseteq \mathbb{F}_{q^m}^{n_i}.$$

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Proposition (U.M. and Zullo (2025))

Let $\mathcal{C}$ be an $[\mathbf{n}, k, d]_{q^m/q}$ sum-rank metric code with constant rank list $(\rho_1, \ldots, \rho_t)$. For any $i \in [t]$, the i-th projection $\mathcal{C}_i$ of $\mathcal{C}$ is a one weight rank-metric code with minimum distance ρ_i.

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Definition

A rank-metric code with parameters $[km, k, m]_{q^m/q}$ is called **simplex rank metric code**.

Theorem (Alfarano, Borello, Neri and Ravagnani (2023))

Let $k \geqslant 2$ and let $\mathcal{C}$ be an $[n, k, d]_{q^m/q}$ one weight code. Then, $n = km$ and $d = m$. That is, $\mathcal{C}$ is isometric to a simplex rank-metric $[km, k, m]_{q^m/q}$ code.

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Theorem (U.M. and Zullo (2025))

Let $\mathcal{C}$ be a non-degenerate $[\mathbf{n}, k, d]_{q^m/q}$ sum-rank code with constant rank list $(\rho_1, \ldots, \rho_t)$. If $k_i = \dim_{q^m}(\mathcal{C}_i) \geqslant 2$, then $\mathcal{C}_i$ is an $[mk_i, k_i, m]_{q^m/q}$ code. If $\dim_{q^m}(\mathcal{C}_i) = 1$, then $\mathcal{C}_i$ is an $[n_i, 1, n_i]_{q^m/q}$ code.

Corollary (U.M and Zullo (2025))

Let $\mathcal{C}$ non-degenerate $[\mathbf{n}, k, d]_{q^m/q}$ sum-rank code with constant rank list $(\rho_1, \ldots, \rho_t)$. For any $i \in [t]$, let $k_i = \mathbf{dim}_{q^m}(\mathcal{C}_i)$. We have that for every $i \in [t]$

$$n_i = mk_i \text{ if } k_i \geqslant 2$$

and

$$\rho_i = \begin{cases} m & \text{if } k_i \geqslant 2, \\ n_i & \text{if } k_i = 1, \end{cases}$$

$\mathcal{I} = \{i \in [t] : k_i \geqslant 2\}$ and $d(\mathcal{C}) = m|\mathcal{I}| + \sum_{i \in [t] \setminus \mathcal{I}} n_i$.

Definition

Let $\mathcal{C} \leqslant \mathbb{F}_{q^m}^n$ and $(\mu_1, \ldots, \mu_t) \in \mathbb{N}^t$ such that $\mu_1 \geqslant \ldots \geqslant \mu_t \implies \mathcal{C}$ is a $(\mu_1, \ldots, \mu_t)$ **constant rank profile** sum-rank metric code if for every $c \in \mathcal{C} \backslash \{0\}$

$$\mu(c) = (\mu_1, \ldots, \mu_t).$$

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Corollary (Neri, Santonastaso and Zullo (2023))

Let $\mathcal{C}$ non-degenerate $[(n,\dots,n),k]_{q^m/q}$ code which is $(\mu_1,\dots,\mu_t)$ constant rank profile. Then

- The number
$$\ell = \frac{t(q^n-1)(q^m-1)}{(q-1)(q^{km}-1)}$$
is a positive integer.
- It holds that
$$tq^{m(k-1)}(q^n-1)(q^m-1) = (q^{km}-1)\left(tq^n - \sum_{i=1}^{t} q^{n-\mu_i}\right).$$

Proposition (U.M. and Zullo (2025))

Let $\mathcal{C} \leqslant \mathbb{F}_{q^m}^n$ and $(\mu_1, \ldots, \mu_t) \in \mathbb{N}^t$. Let $(\mathcal{U}_1, \ldots, \mathcal{U}_t) \leqslant_{\mathbb{F}_q} \mathbb{F}_{q^m}^k$ where $\mathcal{U}_i \leqslant_{\mathbb{F}_q} \mathbb{F}_{q^m}^k$ a system associated with $\mathcal{C}$.

$$\mathcal{C} \text{ is } (\mu_1, \ldots, \mu_t) \textbf{ constant rank profile } \iff$$

$$\tau\Big(n_1 - \dim_{\mathbb{F}_q}(\mathcal{U}_1 \cap v^{\perp}), \ldots, n_t - \dim_{\mathbb{F}_q}(\mathcal{U}_t \cap v^{\perp})\Big) = (\mu_1, \ldots, \mu_t)$$

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Construction

- $\mathcal{C} \rightarrow$ non-degenerate sum-rank metric code

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- $\mathcal{U}_i = \langle \mathbf{x}_i \rangle_{\mathbb{F}_{q^m}} \oplus \mathcal{S}_e^i$
- One dimenional $\mathbb{F}_{q^m}$-subspaces $(\langle \mathbf{x}_1 \rangle_{\mathbb{F}_{q^m}}, \ldots, \langle \mathbf{x}_{q^m+1} \rangle_{\mathbb{F}_{q^m}})$.

Constant Rank Profile

Construction

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- One dimenional $\mathbb{F}_{q^m}$-subspaces $(\langle \mathbf{x}_1 \rangle_{\mathbb{F}_{q^m}}, \ldots, \langle \mathbf{x}_{q^m+1} \rangle_{\mathbb{F}_{q^m}})$.
- $\dim_{\mathbb{F}_q}(S_e^i) = e$, and $\mathcal{S}_e^i \cap \langle \mathbf{x}_\mathbf{i} \rangle_{\mathbb{F}_{\mathbf{q}^\mathbf{m}}} = \{\mathbf{0}\}$

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Proposition (U.M. and Zullo (2025))

$$\mathcal{U}_i = \langle \mathbf{x}_i \rangle_{\mathbb{F}_{q^m}} \oplus \mathcal{S}_e^i, \iff \mathcal{C} \text{ is a } (m, \ldots, m, e) \textbf{ constant rank profile}$$

Theorem (U.M. and Zullo (2025))

$$\mathcal{U}_{ij} = \langle \mathrm{x_i} \rangle_{\mathbb{F}_{q^m}} \oplus \mathcal{S}_e^j \iff \mathcal{C} \text{ is a } (m, \dots, m, e, \dots, e) \text{ constant rank profile}$$

Thank You!