Analysis of Automated Proof of Origami Morley's Theorem

Hidekazu Takahashi, Fadoua Ghourabi, Asem Kasem, and Tetsuo Ida

Department of Computer Science, University of Tsukuba,
Tsukuba 305-8573, Japan
{hidekazu, ghourabi, kasem,ida}@score.cs.tsukuba.ac.jp

Abstract

Morley's theorem states that for any triangle, the intersections of the adjacent angle trisectors form an equilateral triangle. The construction of the Morley's triangle by the straightedge and compass is impossible because of the well known impossibility result of the angle trisection by the straightedge and compass. However, by origami, it is possible to construct an angle trisector, and hence a Morley's triangle.

We have shown previously a stream-lined computer assisted construction and proof of the correctness of the construction by using *Eos* (E-origami system) that we have developed. The proof is based on the Gröbner basis method. A set of polynomials that expresses the premise of the theorem is automatically generated during the construction. This set will be the input to the Gröbner basis computation, together with the set of polynomials describing the conclusion part of the theorem, the basic laws of trigonometry and the condition that filters out ill-formed 9 triangles out of 27 triangles formed by the intersections of the angle trisectors.

It turned out that the Gröbner basis computations took very long time for some origami construction methods. For some cases, the Gröbner basis computations do not terminate successfully because of the lack of memory even on servers equipped with 8 GB of memory. We experimented several origami construction methods and measured the computing time. The computing timings vary greatly depending on the methods of construction, the methods of the coordinate assignment, and on the monomial orders. We analyze the performances by examining the generated polynomial set and report the result of the analysis. We also mention the computing environment that makes the extensive measurements very effective.