

# Exponential Polynomials and Generalized Lambert W Function

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**20 Years of Lambert  $\mathcal{W}$   
Lecture Series**



An exponential polynomial is a function which can be expressed as a polynomial in a variable  $x$  and an exponential function. Finding the real solutions of exponential polynomials has a great interest in many fields such as electromagnetism and quantum mechanics. In this talk, we will present general results on the separation of real solutions and determination of intervals containing all the solutions. The tools used are essentially cylindrical decomposition, Cauchy's bound and Sturm sequences.

Then more specific and formal results are given for exponential polynomials of the form  $\exp(-cx) = a_0(x - r_1)(x - r_2)$ ,  $\exp(-cx) = P(x)$  and  $\exp(-cx) = P(x)/Q(x)$  where  $a_0$ ,  $r_1$  and  $r_2$  are real numbers,  $P$  and  $Q$  are real polynomials. The three previous equations are called generalized Lambert W functions. In the quadratic case ( $\exp(-cx) = a_0(x - r_1)(x - r_2)$ ), the exact number of solutions is given. We determine conditions in order to obtain multiple solutions and closed form solutions are provided in a number of cases. For the more generalized form ( $\exp(-cx) = P(x)$  and  $\exp(-cx) = P(x)/Q(x)$ ) we study counting the number of solutions and an upper bound for the number of real

solution is given. The tools used here are the false derivative and the Lambert W function.

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