Challenges in Multivalued Matrix Functions

Nicholas J. Higham

University of Manchester, School of Mathematics

December 5, 2016

Middlesex College, Room 105B

3:30 - 4:30PM

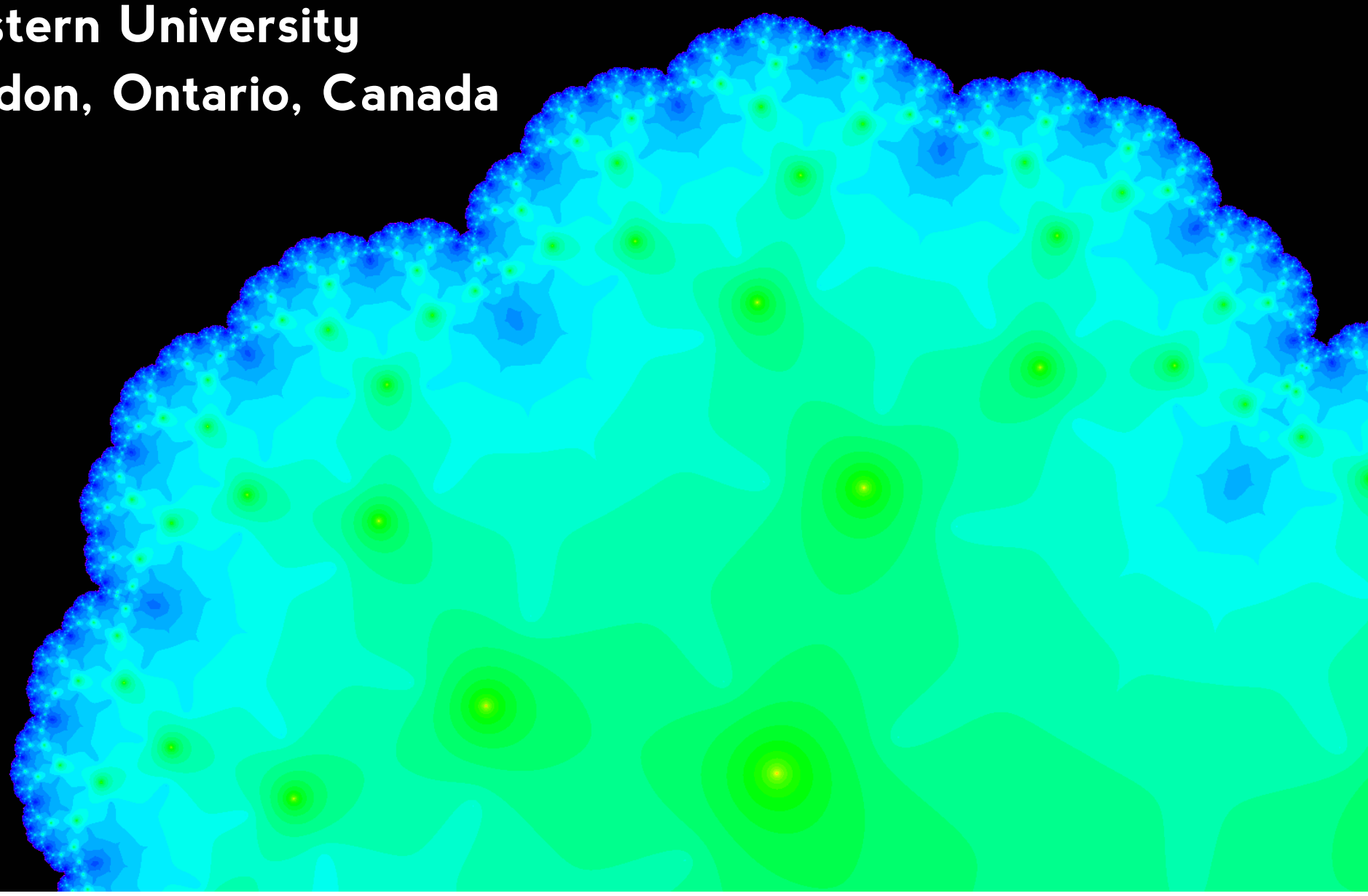
20 Years of Lambert $\mathcal{W}$ Lecture Series

Multivalued matrix functions arise in solving various kinds of matrix equations. The matrix logarithm is the prototypical example. Another example is the Lambert W function of a matrix, which is much less well known but has been attracting recent interest. A theme of the talk is the importance of choosing appropriate principal values and making sure that the correct choices of signs and branches are used, both in theory and in computation. We will give examples where incorrect results have previously been obtained.

We focus on matrix inverse trigonometric and inverse hyperbolic functions, beginning by investigating existence and characterization. Turning to the principal values, various functional identities are derived, some of which are new even in the scalar case, including a "round trip" formula that relates $\operatorname{acos}(\cos A)$ to A and similar formulas for the other inverse functions. Key tools used in the derivations are the matrix unwinding function and the matrix sign function.

A new inverse scaling and squaring type algorithm employing a Schur decomposition and variable-degree Padé approximation is derived for computing acos, and it is shown how it can also be used to compute asin, acosh, and asinh. In numerical experiments the algorithm is found to behave in a forward stable fashion and to be superior to computing these functions via logarithmic formulas.

Western University
London, Ontario, Canada



FIELDS

Maplesoft
Mathematics • Modeling • Simulation
A Cybernet Group Company

ORCCA

Western University · Canada

PROGRAM COMMITTEE

Jonathan Borwein
Rob Corless
David Jeffrey
Ilias Kotsireas
Chris Smeenk