Yukawa effects on the orbital energy of celestial bodies Application to Pulsar PSR1913+16

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Abstract

Research on gravitational theories have involved several contemporary modified models that predict the existence of a non-Newtonian Yukawa-type correction to the classical gravitational potential. In this presentation we consider a Yukawa potential and its effects on the semi-major axis of a two-body system, when the orbiting bodies are in a circular orbit (e=0) and in an elliptical orbit ($0 < e < 1$). Moreover, the change of the semimajor axis over time is calculated and used to solve for different orbital effects including the energy of the system. In addition, we also consider three different scenarios for the circular case where: the semimajor axis is less than the range of interaction; the semimajor axis is of the order of the range of interaction; and the semimajor axis is greater than the range of interaction. In addition, the change in energy with respect to eccentric anomaly as well as the orbital period are calculated for both the circular and elliptical orbits. We have found that the orbital period remains unchanged in the circular orbit case but not in the elliptic case. Lastly, we compare the energies calculated from the circular and elliptic orbital cases to that of the classical Newtonian energy. For the circular orbit case we found that the energy remains unchanged. However, in elliptical orbits, the significance of the Yukawa effect is dependent on the eccentricity of the system as well as the range of the potential λ. A higher eccentricity corresponds to a greater total energy from this effect.

Introduction

Newton's gravitational law of attraction was formulated by a study of the apparent behaviour objects. Although the accuracy of this theory has been proven in many cases, observations however, are not always in agreement with the inverse square law (Haranas et al. 2010). Even using general relativity, although accurate in its predictions, is unsuitable in some cases since realistic celestial systems cannot be considered as isolated/closed systems. Therefore, many present theories of gravitation predict that forces must be tied to gravitation. These forces are expected to be significant for pairs of celestial bodies that lie within a mutual distance greater than 10^{10} m of one another. Studying these forces, that are very small modifications made to Newtonian gravity, use phenomena that build on the original Newtonian potential. Newton himself was one of the first to consider a modified potential of the form where he showed that such a force causes the orbiting bodies to be in a precessional elliptic relative orbit (Newton 1687). Put another way, the relative elliptic orbit of a two-body system has a focal axis that rotates within the plane of motion. In the 20th century a Bulgarian physicist named Georgi Manev proposed a gravitational model based on the same potential above, with specified values for a and b (Manev 1924, 1925, 1930a, 1930b). This model could explain solar system dynamical phenomena with the accuracy of general relativity but without leaving the framework of classical mechanics. There have been many contemporary attempts to modify classical gravity at long astrophysical and cosmological scales. These attempts were motivated by the need to explain observed phenomena that occur at various scales, which at present, have not been given satisfactory explanations in terms of conventional physics (gravitational or not). To name a few, such phenomena include: The Saturn Perihelion (Iorio 2009, 2010), Earth flyby anomalies (Anderson et al. 2007), the Pioneer anomaly (Anderson et al. 1998, 2002). In the 1930's a Japanese theoretical physicist named Hideki Yukawa proposed a new potential energy of the form (Yukawa 1935):

$$V = -\frac{GMm}{r}\left(1+\alpha e^{-\frac{r}{\lambda}}\right)$$

Models that include Yukawa type extra-accelerations have been used for many astronomical situations, that range from solar-system effects (i.e. Pioneer anomaly) to astrophysical and cosmological scenarios. Our focus is to calculate the semimajor axis orbital effects of a Yukawa-type potential in a two body scenario. Within this framework, the change of the semimajor axis (as against the standard Kepler problem) is determined for circular orbits , and use to calculate the energy of the system. Furthermore, we consider the scenario where the primary body loses mass in a specific way is established through the introduction of a mass loss function. Lastly, we consider different cases for the range of the interaction (λ) in relation to the semimajor axis (a), including where: $\lambda >> a$ $\lambda < a$ $\lambda \approx a$

Classical case

We first consider a two-body problem in a circular orbit (e=0), while expressing the motion of a secondary body of mass (m) under the influence of a primary mass (M). With the potential being central, the two-body problem can be reduced to a central-force problem, and the motion of the secondary body can be examined. The Newtonian gravitational potential of the system is then

$$\varepsilon_N = -\frac{GMm}{2a} \quad (1)$$

Where the semimajor axis (a) is set to be equivalent to the orbital radius, G is the gravitational constant. Differentiating this equation with respect to time this equation takes the form of the total orbital power of the system:

Differentiating (1) with respect to time, this equation takes the form of the total orbital power of the system:

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = -\left(\frac{GMm}{a^2}\right)\frac{\mathrm{d}a}{\mathrm{d}t} \quad (2)$$

The change in the semimajor axis with respect to time for the classical case, is given by:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \frac{2e}{n}\left(\frac{\sqrt{1-e^2}\,\sin(E)}{1-e\cos(E)}\right) \quad (3)$$

Where E is the eccentric anomaly, and n is the mean motion. However, in the circular case $e = 0$, so consequently (3) and (2) will both be reduced to zero, meaning there is no change in energy for circular orbits. For Elliptical orbits where ($0 < e < 1$) the total power of the system (2) becomes

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = -\frac{GMme}{a^2 n}\left(\frac{\sqrt{1-e^2}\,\sin(E)}{1-e\cos(E)}\right) \quad (4)$$

Using the relation given in (Haranas et al. 2016), (4) can be expressed as the change in energy with respect to eccentric anomaly:

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}E} = -\frac{GMme}{a^2 n^2}\left(\sqrt{1-e^2}\,\sin(E)\right) \quad (5)$$

The Yukawa potential

We now consider the same two-body system with a modified Yukawa potential energy. The effects of gravity on the secondary mass in the presence of a Yukawa correction can be described by the modified potential energy in the form of total orbital energy below (Haranas et al. 2016).

$$\varepsilon = -\frac{GMm}{2a}\left(1+\alpha\beta^{\frac{a}{\lambda}}\right) \quad (6)$$

α is a coupling constant, λ is the range of the interaction, and $\beta = e^{-1}$ to help differentiate between Euler's constant and the eccentricity. Differentiating this equation with respect to time this equation takes the form of the total orbital power of the system:

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = \frac{GMm}{2a^2}\left(1+\alpha\left(1+\frac{a}{\lambda}\right)\beta^{\frac{a}{\lambda}}\right)\frac{\mathrm{d}a}{\mathrm{d}t} \quad (7)$$

Rate of change equations for circular orbits

In the presence of perturbation, the rates of change of orbital elements in the system undergoing circular motion can be found using Gauss planetary equations, where the semimajor axis component is given by:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = -\frac{2GM\alpha e}{a^2 n}\sin(E)\left(1+\frac{a}{\lambda}\right)\beta^{\frac{a}{\lambda}} \quad (8)$$

Using (8) as well as the relation given in (Haranas et al. 2016), the total orbital power (7) can be expressed with respect to the eccentric anomaly for circular orbits by:

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}E} = -\frac{G^2M^2m\alpha e}{a^4n^2}\sin(E)\left[1+\alpha\left(1+\frac{a}{\lambda}\right)\beta^{\frac{a}{\lambda}}\right]\left[\beta^{\frac{a}{\lambda}}\left(1+\frac{a}{\lambda}\right)\right] \quad (9)$$

Rate of change equations for elliptic orbits

We now consider a two body system that experiences perturbation while undergoing elliptic motion ($0 < e < 1$). We find the total orbital power of the system (7) to be given by:

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} = \frac{GMm}{2a^2(1-e\cos(E))}\left(1+\alpha\left(1+\frac{a}{\lambda}\right)\beta^{\frac{a(1-e\cos(E))}{\lambda}}\right)\frac{\mathrm{d}}{\mathrm{d}t}\left[a(1-e\cos(E))\right] \quad (10)$$

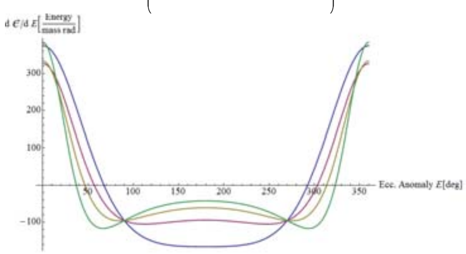

Figure 2 The change in energy with respect to eccentric anomaly for the elliptical case (11), plotted against the eccentric anomaly, where the blue, magenta, green olive, and olive curves correspond to eccentricities (e): 0.2, 0.3, 0.4 and 0.5 respectively

Simplifying (9) and applying the relation given by (Haranas et al. 2016), the total orbital power rate w.r.t. E can be expressed in terms of the eccentric anomaly for elliptic orbits in the following way :

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}E} = \frac{G^2M^2(a+\lambda-ae\cos(E))^2\left[(1+e^2)\cos(E)-2e\right]}{a^5n^2e\lambda^2(e\cos(E)-1)^4}\beta^{\frac{-2a(e\cos(E)-1)}{\lambda}} \quad (11)$$

Integrating (11) for over one revolution the total energy of the system (for $e << 1$) we obtain the following,

$$\begin{aligned}\varepsilon(E) = \varepsilon_0 - \frac{G^2M^2\beta^{\frac{2a}{\lambda}}\pi}{4a^{10}n^2e\lambda^2}\Bigg[ae\Big(&8a^6e^2(10e^6-13e^4 16e^2-21)\\&+4a^5\lambda(72e^6-101e^2+1)+\\&+2a^4\lambda^2(340e^6-459e^4+2e^2-67)+4a^3\lambda^3(250e^4+167e^2-67)+\\&+3a^2\lambda^4(975e^4-1087e^2-8)+1752a\lambda^5(e^2-1)+5250\lambda^6(e^2-1)\Big)\mathrm{BesselI}\left(0,\frac{2\alpha}{\lambda}\right)\\&-\Big(a^7(-128e^8+80e^6+104e^4+8)-4a^6\lambda(80e^8-125e^6+57e^4-42e^2-2)+\\&-4a^5\lambda^2(148e^6-27e^4-101e^2+1)+a^4\lambda^3(-1680e^6+2071e^4-59e^2+134)+\\&-4a^3\lambda^4(469e^4-386e^2-67)-6a^2\lambda^5(925e^4-981e^2-4)-1752a\lambda^6(e^2-1)+\\&-5250\lambda^7(e^2-1)\Big)\mathrm{BesselI}\left(1,\frac{2ae}{\lambda}\right)\end{aligned} \quad (12)$$

Relational effects on the rate of change and orbital period of the system

As mentioned in the introduction we now consider modified equations for the circular case (9), where we take into consideration relational cases, that link the range of interaction to that of the semimajor axis,

Case i) Where $\lambda = a$,

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}E} = -\frac{2\beta G^2M^2\alpha e}{a^4n^2}\sin(E)(1+2\alpha\beta) \quad (13)$$

Case ii) Where $\lambda >> a$,

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}E} = \frac{G^2M^2\alpha e}{n^2a^4\lambda^2}\sin(E)\left(\frac{a^2}{2}(1+2\alpha)-\lambda^2(1+\alpha)\right) \quad (14)$$

Case iii) Where $\lambda < a$,

$$\frac{\mathrm{d}\varepsilon}{\mathrm{d}E} = -\frac{G^2M^2\alpha e}{a^4n^2}\sin(E)\left(1+\frac{a}{\lambda}\right)\beta^{\frac{a}{\lambda}} \quad (15)$$

Using these relations as well as (1) the orbital period of the system is related to the change in energy with respect to eccentric anomaly by:

$$\frac{\mathrm{d}P}{P} = -\left(\frac{3}{2\varepsilon}\frac{\mathrm{d}\varepsilon}{\mathrm{d}E}\right)\mathrm{d}E \quad (16)$$

Where P is the orbital period. Integrating this over one period and solving the resulting equation we find that for the circular case, the orbital period remains unchanged regardless of the case,

$$P = P_0 \quad (17)$$

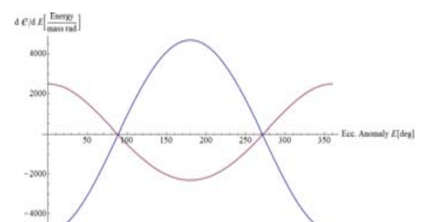

Figure 2 The change in energy with respect to eccentric anomaly for the elliptical case (11), plotted against the eccentric anomaly, where the magenta and blue curves correspond to eccentricities(e): 0.02, and 0.01 respectively.

Numerical results

To proceed with our numerical calculation we assume the two body system is a binary pulsar system, PSR 1913+16, which has the following orbital parameters. $M = M_0 = m = 1.38\ M_\odot$, $a = 1.9501 \times 10^9$ m, $\alpha = 2.40 \times 10^{-8}$, $\lambda = 3.97 \times 10^8$ m, and $n = 8.07 \times 10^{-8}$ rad/s (Deng et al. 2012). For the circular case ($e = 0$), we find that the total energy of the system does not change given a Yukawa potential of the form (7) which agrees with the classical case (3) due to the dependence on eccentricity. Consequently, this also means the orbital period of the system remains unchanged for circular orbits (16).

In the numerical calculations of the elliptical case, the pulsar PSR 1913+16 is again used which has all the same values from the circular case with the exception of eccentricity which has a value of $e = 0.617$. Solving for the change in energy of the system with respect to eccentric anomaly we obtain (11). Figures 1 and 2 plot these values at specific eccentricities where table 1 highlights values at precise angles of E. Evaluating the total energy for the elliptical case (12) using the specific values from PSR 1913+16 ($e = 0.617$) we obtain $\varepsilon(E) = \varepsilon_0 - 2.20\times10^{12}$ Where ε_0 is the initial energy (1). Similarly if we instead use the same numerical values as before while taking the eccentricity to be $e = 0.8$, then from (12) we obtain, $\varepsilon(E) = \varepsilon_0 - 4.40\times10^{12}$. We then conclude that the Yukawa effect is more significant as the eccentricity of the system increases due to the decreasing size of the periastron (closest point between two bodies in an orbit) for increasing eccentricity.

Table 1 Numerical values for the change in energy with respect to eccentric anomaly, where specific eccentricities are evaluated at a range of eccentric anomalies, column 5 corresponds to the pulsar PSR 1913+16

	$\frac{\mathrm{d}\varepsilon}{\mathrm{d}E} = \frac{G^2M^2(a+\lambda-ae\cos(E))^2\left[(1+e^2)\cos(E)-2e\right]}{a^5n^2e\lambda^2(e\cos(E)-1)^4}\beta^{\frac{-2a(e\cos(E)-1)}{\lambda}}$ [J s^{-1} kg^{-1}]			
Eccentric Anomaly E [°]	Eccentricity $e = 0.01$	Eccentricity $e = 0.02$	Eccentricity $e = 0.40$	Eccentricity $e = 0.617$
0	4893.28	2496.83	333.05	529.892
45	4892.79	2496.56	332.86	529.10
90	4891.33	2495.77	332.31	526.74
180	4885.48	2492.61	330.11	517.37
360	4862.11	2479.99	321.42	481.0

Conclusions

Using a corrected potential in the form of a Yukawa potential is largely dependent on the eccentricity of the orbit. As was shown for circular orbits, a Yukawa potential doesn't effect the total energy or period a two body system that is undergoing circular motion ($e = 0$). In the presence of an elliptical ($0 < e < 1$) orbit, the Yukawa's effect on a two body system is more significant with increasing eccentricity. This is due to the fact that as the eccentricity of the orbit increases the periastron decreases proportionally. Moreover, the closer the approach of these two bodies, the greater the proportion of energy that comes from Yukawa effect.

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