The Effect of Coulomb Drag on the Mean Motion of Low Earth Orbit Satellites

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Abstract

Our world has been forever changed by the presence of satellite technology. From GPS to the cellular telecommunications network, to advanced satellites such as Hubble which have transformed the way we see ourselves in the universe and how we communicate with each other. Thus an obvious issue is to quantify and readjust the drag. This has been done in various papers.

The problem addressed here is the drag of LEO (low earth orbit) satellites cause by columbic interactions in the upper atmosphere of Earth. Both theoretical and numerical predictions are made and should be measureable. The approximate size, eccentricity, and other parameters are all well known . We can use these to find the numerical results and see if how significant of an effect these forces of drag have on the mean motion of these satellites. This was done in a paper of Lin-Sen Li [2], however mean motion was not done instead many other orbital parameters were looked at.

Introduction

Since the USSR launched the first satellite Sputnik around the world on October 4, 1957, the technology behind satellite technology has grown at an enormous rate. The positioning of these satellites are of crucial importance no matter the type of satellite. A famous example of why satellite positioning is so crucial is when the GPS system was first created the engineers working on the project turned the correction system which would correct the relativistic effects. Although these effects are relatively small but quickly these effects caused the positioning system to be completely off. There are many possible effects which can change the positioning of satellites and LEO satellites are particularly susceptible to drag phenomena because of the interaction with the upper atmosphere.

The size of satellites will as one might suspect play a major role in how much drag is applied. Thus the recent surge of development of nano-satellites may help to mitigate some these effects. However, the majority of LEO satellites which are currently being used are of the size of about 0.5-1.0 meters in diameter. This higher cross sectional surface area will cause more drag. However, clearly altitude will also have a major effect on drag as the altitude will have a major effect on the density of particles which interact with the satellite. The coulomb drag is produced by the Coulomb force between the charge on the satellite and charge on the ions, which we will consider to be electrons as they will be the primary attributer to this phenomenon. It was Chopra [4] who originally pointed out that Coulomb forces with interactions of the electric field of the satellite changes the motion of such satellites. Jastrow and Pearse [3], Kraus and Watson [7], all developed on these ideas but none looked at the secular effect of these forces.

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Classical Gravitation al Two Body Problems

The theoretical results all stem from the classical gravitational two body problem. The two bodies are clearly the earth and satellite [1].

$$GM = a(t)^2 n(t)^3 \quad (1)$$

Then, taking the time derivative can be taken and by doing some rearranging of the equation the result becomes [1].

$$\frac{dn}{dt} = -\frac{3\,n(t)}{2\,a(t)}\frac{da}{dt} \quad (2)$$

This is a widely recognized result seen in Haranas 2010 [1] . This equation allows us to represent the change in mean motion with respect to time as related to the change in semi major axis in time by a time dependent function. This, can then by a simple substitution of Gauss' relations which are as follows. This substitution will later yield the theoretical model for columbic drag of low earth orbit satellites [2].

$$\frac{da}{dt} = \frac{n(t)a(t)^2}{(1-e^2)} C_{D,e} \frac{(\pi R^2)}{m_s} m_e N (1+e^2+2e\cos(f))^{\frac{3}{2}} \quad (3)$$

The Predicted Effects of Coulomb Drag on the Mean Motion of Low Earth Orbit Satellites

From the previous section we know important equations which are useful for many parameters of objects in orbit. By a simple substitution of equation 3) into equation 1) yields a differential equation which will become the main predictive equation. However, it is also necessary to do a transform of the fundamental anomaly with the eccentric anomaly. This transformation is done with the following transformation [1].

$$\cos(f) = \frac{\cos(E) - e}{1 - e\cos(E)} \quad (4)$$

This will allow us to express the change of mean motion where the variable parameter is the eccentric anomaly. The eccentric anomaly will be a parameter in radians and for one full revolution will move from 0 to 2π, or a transformation can be done which will change it from 0 to 360 degrees. By doing this substitution we will yield a rather large and cumbersome equation which will have the eccentricity (e) expressed as a cubic equation where we can omit the third order term, this is because the eccentricity is a value less then 1 so the cubed is much less then 1. This now will make out result more simplified while sacrificing negligible accuracy. However, this form will be a differential with respect to time, where it is more valuable to have it as a variable with respect to the eccentric anomaly. In order to this it is necessary to another transformation which will be the following [2].

$$\frac{dE}{dt} = \frac{na}{r} = \frac{n}{1 - e\cos(E)} \quad (5)$$

The result after this substitution will be a linear separable differential equation which will have the limits of integration between 0 and 2π, which has the following form.

$$\int \frac{dn}{n} = \frac{-3a\pi C_{D,e} N m_e R^2}{2m_s\sqrt{1-e^2}} \int_0^{2\pi} \left(1 + \frac{e}{1-e^2}\cos(E)\right) dE \quad (6)$$

By solving this relatively simple equation we yield the following result., which is our theoretical result.

$$\frac{n(E)}{n_0} = e^{\frac{-3aC_{D,e} N m_e (\pi R)^2}{m_s\sqrt{1-e^2}}} \quad (7)$$

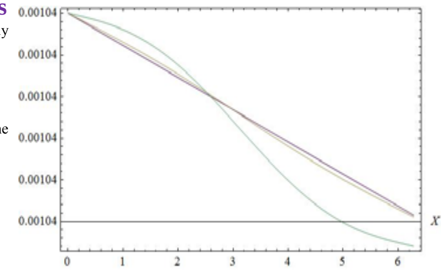

Figure 1. The change of mean motion LEO satellites for one full revolution at 250km in altitude.

Numerical Results

We will take the values of eccentricity of 0, 0.01, 0.13, and 0.5. A 0 eccentricity will represent the orbit of a satellite which is circular, and 0.01 is nearly circular. The eccentricity of 0.13 represents a very usual eccentricity of typical low earth orbit satellites. The eccentricity of highly elliptic satellites. We will also use parameters of radius of the satellite as 0.5 meters which is a very typical size of many modern day satellites. However, a satellite such as Hubble which is the size of a school bus is much bigger, but this is not the case we are considering which are usually smaller communication satellites. .We will consider several values for the altitude which are 250km, 500km, 1000km, and 20,000km in altitude [2]. This corresponds to a semi major axis of 6671km, 7171km, 7671km, and 26621km respectively. Other parameters which are used in these calculations are the mass of the satellite which is denoted by m_s this will be 1000 kg which is average size of most low earth orbit satellites. Then is also the mass of the electron m_e which is $9.11\times10^{-31} kg$ [5]. The last of the values which are relevant to the numeric calculations is the density of electrons in the range of semi major axis' which we are finding solutions for, this value is denoted by N we know the value is of the order of 10^{-11}[2].

Table 1 Values for the Drag Coefficient with Respect to Altitude [2]

Height	250km	500km	10000km	20,000km
Height (km)	250	500	800	2×10^4
$C_{d,I}$	7×10^{-5}	0.32	6.1	7.7×10^3
$C_{d,e}$	2.6×10^{-5}	0.5	1.38	1.1×10^3

Table 2 The Numeric results for mean motion per revolution for eccentricity $e = 0.13$ at various orbital altitudes.

Height	250km	500km	1000km	20,000km
a_0 (km)	6621	6871	7171	26371
$n(E)/n_0$	0.99921	0.9999999	1.0	1.0

Conclusions

To conclude, by using the well known gravitational two body problem and Gauss' Planetary equations it is possible to predict the orbit of an low orbit earth orbit satellite, and its effect on its semimajor axis. The work done in this paper predicted a simple first order linear differential equation for the mean motion of the low orbit satellite, that resulted to a decaying exponential solution. Larger satellites may play a role, in how Coulomb drag effects the mean motion of satellites. However, the eccentricity of the orbit and the orbital altitude are the orbital elements that significantly affect affecting the Coulomb drag.

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