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Implications of Information Theory on the Expansion of the Universe Using Planck Units and the Hubble Constant

Introduction

The smallest unit of information is the bit, a binary unit that has a value of either 0 or 1. In computer science, this often corresponds to the state of an object, which is either high or low, for example, the state of an individual pixel can be described as being either on or off. In other words, the state of this pixel can be described using one bit of information. Furthermore, if one were to flip a coin, only one bit of information would be needed to describe the result of flipping the coin, a 0 could indicate tails and a 1 could indicate heads. The Bekenstein Bound, which is derived in the next section, was discovered by Jakob Bekenstein and provides an upper bound on the information needed to describe a physical system contained in a sphere of radius R down to the quantum level. The Bekenstein Bound has been of particular interest to astrophysicists and cosmologists, most notably Stephen Hawking, who found that the information required to describe black holes is exactly equal to the Bekenstein Bound. This project examines the Bekenstein Bound in terms of Planck units and the Hubble constant and the resulting implications.

Derivation of the Bekenstein Bound

Information theory traces its origins back to Ludwig Boltzmann and his formula for entropy, given as

$$S = k_B \ln(W)$$

where S is the entropy, k_B is the Boltzmann constant and W is the thermodynamic probability. In information theory, a near identical formula was proposed by Claude Shannon and is given as

$$N = k_B \ln(p)$$

where N is the information of the system, k_B is the Boltzmann constant and p is the complexity of the system. The Bekenstein bound in the universal form of a given system was found by Jakob Bekenstein to be,

$$S \leq \frac{2\pi k_B R E}{\hbar c}$$

where S is entropy, k_B is the Boltzmann constant, R is the radius of a sphere that can contain the system, E is the energy of the system, $\hbar$ is the reduced Planck constant and c is the speed of light in a vacuum. Using the formula for the entropy of information,

$$S = N k_B \ln(2)$$

where N is the number of bits of information needed to describe the system being studied and substituting it for the entropy in the Bekenstein bound yields,

$$N k_B \ln(2) \leq \frac{2\pi k_B R E}{\hbar c}$$

$$N \leq \frac{2\pi R E}{\hbar c \ln(2)} \qquad (1)$$

Planck Units

Planck units are units of measurement designed to normalize quantities in terms of natural constants such as c, $\hbar$, G, ϵ_0 and k_B. Given below are values of the Planck time, t_P, and Planck energy, E_P.

$$t_P = \sqrt{\frac{\hbar G}{c^5}}$$

$$E_P = \sqrt{\frac{\hbar c^5}{G}}$$

The above equations can be rearranged to give the following,

$$t_P^2 = \frac{\hbar G}{c^5} \rightarrow G = \frac{t_P^2 c^5}{\hbar}$$

$$E_P^2 = \frac{\hbar c^5}{G} \rightarrow G = \frac{\hbar c^5}{E_P^2}$$

Equating the above, we find,

$$\frac{t_P^2 c^5}{\hbar} = \frac{\hbar c^5}{E_P^2}$$

$$t_P^2 E_P^2 = \hbar^2$$

$$\hbar = t_P E_P \qquad (2)$$

Information in a "Planck Universe"

Replacing E in the Bekenstein Bound with Einstein's $E = mc^2$, yields a Bekenstein Bound given as

$$N \leq \frac{2\pi c R m}{\hbar \ln(2)} \qquad (3)$$

Considering a universe with a radius of l_P and a mass of m_P, the Bekenstein Bound of this "Planck Universe" can be expressed as

$$N \leq \frac{2\pi c l_P m_P}{\hbar \ln(2)}$$

Solving the above equation, the maximum information required to describe a "Planck Universe" is $N \cong 9.06 \equiv 9\ bits$. Identical results were found using other Planck units.

The Bekenstein Bound and the Hubble Constant and the Resulting Implications

Using Equations (1) and (2), the Bekenstein Bound can be rewritten as

$$N \leq \frac{2\pi R E}{t_P E_P c \ln(2)} \qquad (4)$$

Replacing R in the above equation with the Hubble Length, given as, $Hubble\ Length = \frac{c}{H_0}$ yields,

$$N \leq \frac{2\pi E}{t_P E_P H_0 \ln(2)} \qquad (5)$$

Equation (5) is an interesting equation as it is heavily dependent on natural constants and has only two variables, energy, E and the Hubble Constant, H_0. Furthermore, E and E_P can be eliminated and replaced with a constant value. Equation (5) also has interesting potential implications. In the preceding section, in the "Planck Universe" E would be replaced with E_P allowing the upper bound of H_0 to be found in the hypothetical universe. Equation (5) appears to further imply that in order for N not to decrease over time, which unimpeded could spell disaster for the Universe, H_0 must decrease or E must increase or some combination of both scenarios must occur. If the situation is such that E is constant, then in order for N to increase, H_0 must decrease, which would imply that if the Universe is expanding, then said expansion would have to slow. Finally, another potential implication of Equation (5) is its possible value in determining the current value of the Hubble Constant, which has been a major research topic in cosmology in the 21st century. If the Bekenstein Bound of the Universe could be determined reliably, then Equation (5) could be easily used to determine an upper bound for the current Hubble Constant, H_0. Additionally, in the case of black holes where the Bekenstein Bound is saturated, the strict equality could possibly be used to draw even more definitive conclusions.

Conclusion

The Bekenstein Bound is of particular interest to astrophysicists and cosmologists due to the profound insights that it can provide into the current state of the Universe as well as the evolution of the Universe. The Bekenstein Bound and its various manifestations provide valuable governing equations that when combined with astronomical observations, could potentially be used to deepen humanity's understanding of the Universe, the laws governing its expansion and the mechanisms by which such expansion occurs.