Quantum Computation for D-Optimal Designs of Circulant Type

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Abstract

Our research shows the significant computational speedup and potential of using quantum annealing approaches to solving autocorrelation functions. This research shows the prototype and scalability of our quantum algorithm for solving two autocorrelation functions that, when summed, equal two. This problem is known to be NP-hard, but harnessing QC techniques can compute solutions in polynomial time. This poster hopes to give a rudimentary understanding of quantum advantage.

Introduction

- A D-Optimal design promises to provide a clear path to quantum advantage, running this NP-hard problem to make an impact on dozens of fields.
- D-optimal matrices of circulant type can lead to optimizations in industrial processes, the design efficient experiments in the pharmaceutical industry, help us study the environment for pollutants, food production, and more.
- With periodic autocorrelation functions of values {1, -1} we can search for bit strings containing unseen patterns in seemingly stochastic data.

The **periodic autocorrelation function** associated to a finite sequence $A = [a_0, \ldots, a_{n-1}]$ of length n is defined as

$$P_j(s) = \sum_{k=1}^{n} a_k a_{k+s}, \quad s = 0, \ldots, \frac{n-1}{2},$$

where $k+s$ is taken modulo n, when $k+s \geq n$.

Let n be an odd integer, such that $x^2 + y^2 = 4n - 2$ has solutions (α, β), i.e. $\alpha^2 + \beta^2 = 4n - 2$. A **D-optimal design of circulant type** consists of two sequences of length n each, $A = [a_0, \ldots, a_{n-1}]$ and $B = [b_0, \ldots, b_{n-1}]$ with elements from $\{-1, +1, \}$ such that:

$$a_1 + \ldots + a_n = \pm\alpha$$
$$b_1 + \ldots + b_n = \pm\beta$$
$$P_A(s) + P_B(s) = 2, \forall\ s$$

Quantum Computing

- Quantum computing is a new approach to computation that harnesses the properties of quantum mechanics to provide enormous advantages for specific tasks.
- By leveraging the concepts of superposition and entanglement, a quantum computer can effectively perform calculations on all answers simultaneously.

Methodology

Execution of the Algorithm:

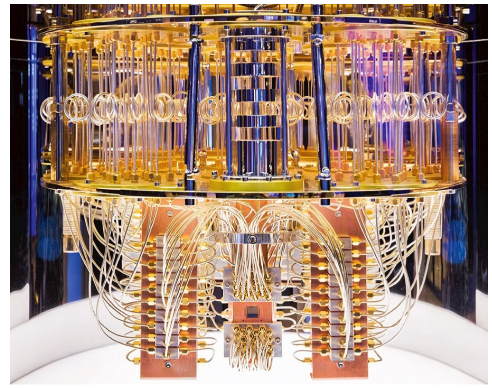
- Running of this algorithm was primarily done through the D-Wave Leap system using Ocean SDK, an open source classical to quantum toolkit.
- We are able to input our equations and constraints into the Quantum Processing Unit (QPU) in a scalable manner.

Problem Formulation:

- There are multiple methods of formulating a problem to be suitable for a quantum computer. Our chosen formulations were Quadratic Unconstrained Binary Optimization (QUBO) and Ising Model.
- These formulations can be directly run on quantum annealing hardware.

Limitations:

- The limitations to current quantum computing are errors, and connectivity.
- While one may expect a computer to always give a correct and consistent answer, this is not the case in quantum. It is entirely possible that our algorithm gives an absurd answer.
- Connectivity is our greatest current limitation, and we hope to develop more algorithms to supplant this problem.
- By creating 'virtual' qubits, we break our problem into many smaller ones and entangle these smaller problems in order to find full solutions.



Results

- Experimentation showed current quantum annealing hardware can quickly produce results for N=33. 33 may sound underwhelming, but it's not. It means there are 2^{33} possible elements in the search space. The quantum computer was able to go through 8.5 billion calculations in 32 milliseconds! As we scale further, errors start to occur in our output.
- This is natural for a quantum computer, as outcomes are probabilistic as opposed to deterministic in the classical world.
- Some amount of improvement is gained by increasing annealing samples, but mostly achieves finding multiple solutions for lower values of N.
- Through our research, we discovered a limitation in the connectivity of qubits. This currently stands at 15, where the total number of usable qubits is 5000+, depending on the QPU in use.

N	QPU Time (In microseconds)	Result Ac.	Qubits	Solutions
3	31686	1.0	6	91
5	31726	1.0	10	108
7	31805	1.0	14	100
15	31822	1.0	30	30
31	31857	1.0	62	1
33	31856	1.0	66	1
63	31854	0.48	126	0
153	16043	0.11	306	0
199	32092	0.23	398	0

Conclusion

The improvements made using quantum computation are leaps and bounds better than the best classical approaches that are available today. We see this scaling in the future with new advancements. Advancements such as D-Wave's next generation Zephyr QPU, which allows more qubits and greater connectivity between its qubits. Advancements in the quantum entanglement of the qubits will shorten the time and computing power used to process the information sharing between qubits. Betterment in quantum noise reduction will preserve accuracy and increase the probability of the calculations that are being performed. All in all, our research proves that there are ground-breaking developments happening in the field of quantum computing, and we are only just getting started.

Recommendations

The highest commercially available qubit connectivity is IBM Osprey with 27. Further trials on these systems may be warranted as QC does not yet have a single system approach to solving problems.

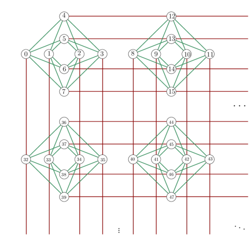


Figure 1

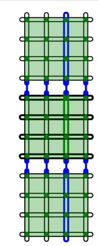


Figure 2

- Figure 1: Depicts the connections between qubits in a graph format.
- Figure 2: An illustration of Josephson's junctions on the QPU and their external couplings.

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